

# **FINITE-DIFFERENCE MODEL OF TWO-DIMENSIONAL, SINGLE-, AND TWO-PHASE HEAT TRANSPORT IN A POROUS MEDIUM—VERSION I**

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U.S. GEOLOGICAL SURVEY  
Open-File Report 77-234

1977



**UNITED STATES DEPARTMENT OF THE INTERIOR**  
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Finite-Difference Model of Two-Dimensional,  
Single-, and Two-Phase Heat Transport  
in a Porous Medium——Version I

by

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**ABSTRACT**

Model documentation is presented for a two-dimensional (areal) heat-transport model capable of simulating both water- and vapor-dominated geothermal reservoirs that conform with the assumptions of the model. Finite-difference techniques are used to solve for the dependent variables pressure and enthalpy. The program is designed to simulate time-dependent problems such as those associated with geothermal reservoirs undergoing exploitation, and can treat the transition from compressed water to two-phase flow. In order to simulate more complicated field problems the present program is being extended, and therefore the model described in this report is referred to as VERSION I. A listing of the computer code is included.

## INTRODUCTION

The continuity equations for steam and water are reduced to two nonlinear partial differential equations in which the dependent variables are fluid pressure and enthalpy. These equations are approximated using finite-difference techniques and are solved using a direct matrix technique. The nonlinear coefficients are calculated using Newton-Raphson iteration on the accumulation terms, and an option is provided for using either upstream or midpoint weighting on the mobility terms. The model can simulate flow of compressed water, two-phase mixtures, and super-heated steam over a temperature range of 10° to 300°C. In addition, it can handle the conversion from single-phase flow to two-phase flow.

The model described in this report is referred to as VERSION I and is considered to be a research tool; that is, this version is kept as simple as possible so that the program can be easily understood and modified. For this reason, many sophisticated changes and additions which are generally required to simulate complicated field problems are not included. Such changes are the subject of current research and will be described in subsequent reports.

## MODEL DESCRIPTION

### Theoretical Development

The computer program presented in this report is based on a theoretical development originally outlined by Mercer, Faust, and Pinder (1974) and presented in detail by Faust (1976). Only the final equations and major assumptions are given here.

Major assumptions include the following:

- 1) capillary pressure is negligible;
- 2) local thermal equilibrium exists among all phases;
- 3) Darcy's equations for two-phase flow are valid;
- 4) the thermodynamic properties of the geothermal fluid are those of a pure-water system;
- 5) thermal dispersion/conduction is treated as a property of the medium;
- 6) flow in the reservoir can be represented by using a two-dimensional (areal) model; and
- 7) the reservoir is a confined porous medium.

## Final Equations

Based on these assumptions the equations describing flow and heat transport in a geothermal reservoir are (Faust, 1976):

$$\nabla \cdot \left( \frac{b\rho_s k k_{rs}}{\mu_s} \nabla p \right) + \nabla \cdot \left( \frac{b\rho_w k k_{rw}}{\mu_w} \nabla p \right) + b q'_s + b q'_w = b \frac{\partial (\phi_p)}{\partial t} \quad (1)$$

and

$$\begin{aligned} \nabla \cdot \left( \frac{b\rho_s h_s k k_{rs}}{\mu_s} \nabla p \right) + \nabla \cdot \left( \frac{b\rho_w h_w k k_{rw}}{\mu_w} \nabla p \right) + \nabla \cdot \{ K_m b [(\frac{\partial T}{\partial p})_h \nabla p + (\frac{\partial T}{\partial h})_p \nabla h] \} \\ + b q''_s h'_s + b q''_w h'_w + q'' = b \frac{\partial}{\partial t} [\phi \rho h + (1-\phi) \rho_r h_r] \end{aligned} \quad (2)$$

where the term,  $q''$ , represents the conductive-heat gain (or loss) to the confining beds (overburden and underburden), and may be obtained from,

$$q'' = K \frac{\partial T}{r \partial z} \Big|_{\substack{\text{overburden} \\ \text{contact}}} - K \frac{\partial T}{r \partial z} \Big|_{\substack{\text{underburden} \\ \text{contact}}} \quad (3)$$

Definitions of the parameters are included in the section on notation.

## Auxiliary Relationships

Additional assumptions and relationships include the following:

- 1) The fluid enthalpy,  $h$ , of the mixture is defined in the two-phase region by:

$$h = \frac{S_S \rho_S h_S + S_W \rho_W h_W}{\rho} . \quad (4)$$

- 2) The density,  $\rho$ , of the mixture is defined by

$$\rho = S_S \rho_S + S_W \rho_W . \quad (5)$$

- 3) Phase saturations sum to one:

$$S_S + S_W = 1 . \quad (6)$$

- 4) Porosity is a function of pressure, and can be expanded about an initial porosity distribution by a truncated Taylor series with first order pressure terms:

$$\phi = \phi_i [1 + \beta(p - p_i)] , \quad (7)$$

where the subscript  $i$  indicates initial values.

- 5) Phase viscosities are functions of temperature (Meyer and others, 1967; modified for the cgs system),

$$\mu_S = 10^{-6}(0.407 \cdot T + 80.4) , \quad (8)$$

and,

$$\mu_w = 10^{-6} \left\{ 241.4 \cdot 10^{[247.8/(T+133.15)]} \right\}, \quad (9)$$

where,

$\mu_s$  = dynamic viscosity of steam, g/cm-sec,

$\mu_w$  = dynamic viscosity of water, g/cm-sec,

T = temperature, °C.

Equation 8 is valid for superheated steam at  $10^6$  dynes per square centimeter pressure in the temperature range of 100 to 300 degrees Celsius, and is approximately valid for steam viscosity along the saturation line in that range. Equation 9 is valid for liquid water along the saturation line from 0 to 300 degrees Celsius.

6) The relative permeability expressions are functions of saturation and are a variation of those given by Corey (1954) for a drainage displacement process, that is, vaporization dominates condensation:

$$k_{rw} = \frac{(s_w - s_{wr} - s_{sr})^4}{(1 - s_{wr} - s_{sr})^4}, \quad (10)$$

and

$$k_{rs} = \left[ 1 - \frac{(S_w - S_{wr} - S_{sr})}{(1 - S_{wr} - S_{sr})} \right]^2 . \quad (11)$$

$$\left[ 1 - \frac{(S_w - S_{wr} - S_{sr})^2}{(1 - S_{wr} - S_{sr})^2} \right] ,$$

where  $S_{wr}$  and  $S_{sr}$  are specified residual water and steam saturations.

7) Reservoir thickness, rock density, rock specific heat and intrinsic permeability are functions of the spatial coordinates.

8) Rock enthalpy may be determined by the expression,

$$h_r = c_r T \quad (12)$$

where the rock enthalpy is in ergs per gram, the temperature is in degrees Celsius, and the formation heat capacity,  $c_r$ , is in ergs per gram per degree Celsius.

9) In the two-phase region, the amount of heat lost to the well is defined as

$$q_h = q'_s h'_s + q'_w h'_w , \quad (13)$$

and the total mass lost to the well as,

$$q_m = q'_s + q'_w . \quad (14)$$

The steam production rate is determined by the fractional flow of the steam phase as follows,

$$q'_s = \sigma_s q_m , \quad (15)$$

where

$$\sigma_s = k_{rs}/(k_{rs} + \frac{\rho_w \mu_s}{\rho_s \mu_w} k_{rw}).$$

Since  $h'_s$ ,  $h'_w$  and  $q_m$  are known,  $q'_s$  is computed using equation 15,  $q'_w$  is computed using equation 14, and  $q_h$  is computed using equation 13. Finally, equations relating the thermodynamic properties of pure water and steam to enthalpy and pressure were determined by least square regressions of published experimental data. Figure 1 shows a pressure-enthalpy plot for pure water, this diagram may be divided into the subregions: compressed water, two-phase (steam-water), and superheated steam. Data were obtained from Meyer and others (1967) and Keenan and others (1969) for an enthalpy range of  $2.09 \times 10^9$  to  $3.175 \times 10^{10}$  ergs per gram, a pressure range of  $1.0 \times 10^6$  to  $1.75 \times 10^8$  dynes per square centimeter, and a temperature range of 10 to 300 degrees Celsius. The following expressions, having a maximum error of 0.5 percent within the observed data range, were determined:

- 10) Steam enthalpy,  $h_s$ , and water enthalpy,  $h_w$ , are treated as functions of pressure:

$$h_s = 2.82282 \cdot 10^{10} - 3.91952 \cdot 10^{15}/p \\ + 2.54342 \cdot 10^{21}/p^2 - 9.38879 \cdot 10^{-8} \cdot p^2 \quad (16)$$

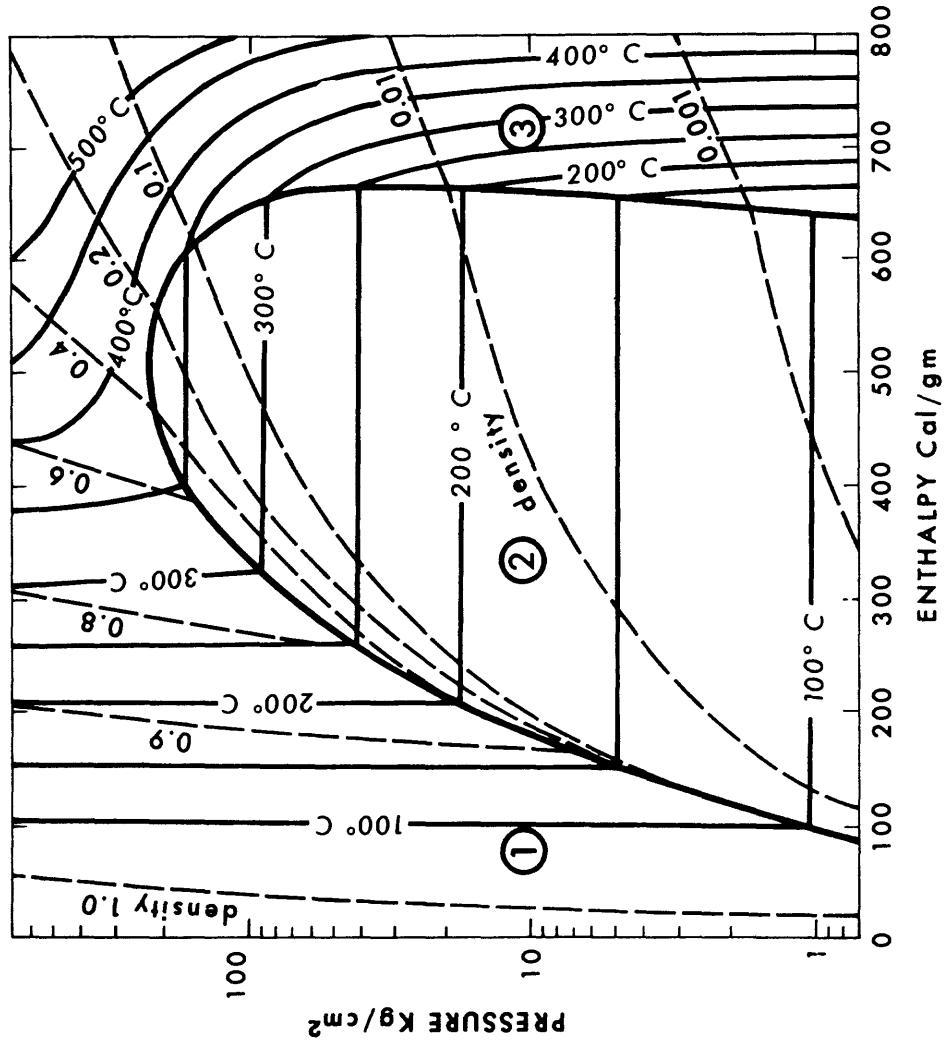


Figure 1. Pressure-enthalpy diagram for pure water and vapor showing three thermodynamic regions below the critical point: 1) compressed water, 2) two-phase steam and water and 3) superheated steam (modified from White, Muffler, and Truesdell, 1971).

and

$$\begin{aligned} h_w = & 7.30984 \cdot 10^9 + 1.29239 \cdot 10^2 p - 1.00333 \\ & 10^{-6} \cdot p^2 + 3.9881 \cdot 10^{-15} \cdot p^3 \\ & - 9.90697 \cdot 10^{15}/p + 1.29267 \cdot 10^{22}/p^2 \\ & - 6.28359 \cdot 10^{27}/p^3, \end{aligned} \quad (17)$$

where

$p$  = pressure, dynes/cm<sup>2</sup>

$h$  = enthalpy, ergs/g

$h_s$  = enthalpy of saturated steam, ergs/g

$h_w$  = enthalpy of saturated water, ergs/g.

11) Temperature is treated as a function of pressure and enthalpy for the compressed-water and superheated-steam regions. For the compressed-water region,

$$\begin{aligned} T = & - 2.41231 + 2.56222 \cdot 10^{-8} \cdot h \\ & - 9.31415 \cdot 10^{-17} \cdot p^2 \\ & - 2.2568 \cdot 10^{-19} \cdot h^2, \end{aligned} \quad (18)$$

and for the superheated-steam region,

$$\begin{aligned} T = & -374.669 + 4.79921 \cdot 10^{-6} \cdot p \\ & - 6.33606 \cdot 10^{-15} \cdot p^2 \\ & + 7.39386 \cdot 10^{-19} \cdot h^2 \\ & - 3.3372 \cdot 10^{34} / h^2 p^2 \\ & + 3.57154 \cdot 10^{19} / p^3 \\ & - 1.1725 \cdot 10^{-37} \cdot h^3 p \\ & - 2.26861 \cdot 10^{43} / h^4, \end{aligned} \tag{19}$$

where the temperature,  $T$ , is in degrees Celsius. For the two-phase (steam-water) region  $h_w$  is used in place of  $h$  in equation 18.

12) Total density,  $\rho$ , steam and water densities,  $\rho_s$  and  $\rho_w$  are considered functions of pressure and enthalpy. For the compressed-water region,

$$\begin{aligned} \rho = \rho_w = & 1.00207 + 4.42607 \cdot 10^{-11} \cdot p \\ & - 5.47456 \cdot 10^{-12} \cdot h \\ & + 5.02875 \cdot 10^{-21} \cdot hp \\ & - 1.24791 \cdot 10^{-21} \cdot h^2, \end{aligned} \tag{20}$$

and for the superheated-steam region,

$$\begin{aligned}\rho = \rho_s = & - 2.26162 \cdot 10^{-5} + 4.38441 \cdot 10^{-9} \cdot p \\ & - 1.79088 \cdot 10^{-19} \cdot ph \\ & + 3.69276 \cdot 10^{-36} \cdot p^4 \\ & + 5.17644 \cdot 10^{-41} \cdot ph^3 ,\end{aligned}\tag{21}$$

where density is in grams per cubic centimeter. For the steam-water region, saturation pressures and enthalpies are used in equation 20 and 21 to obtain  $\rho_s$  and  $\rho_w$ .

## Numerical Development

The technique used to solve equations 1 and 2 is based on the finite-difference method. For this method the areal extent of the reservoir is subdivided into rectangular grid blocks (see figure 2) in which the fluid and reservoir properties are assumed uniform. The continuous derivatives in equations 1 and 2 are approximated by finite-difference expressions at points (nodes) in the centers of the blocks. This results in a nonlinear system of  $2n$  equations with  $2n$  unknowns (the values of pressure and enthalpy at the nodes) where  $n$  is the number of nodes. The general finite-difference representation and solution procedure for this system of nonlinear equations are outlined below.

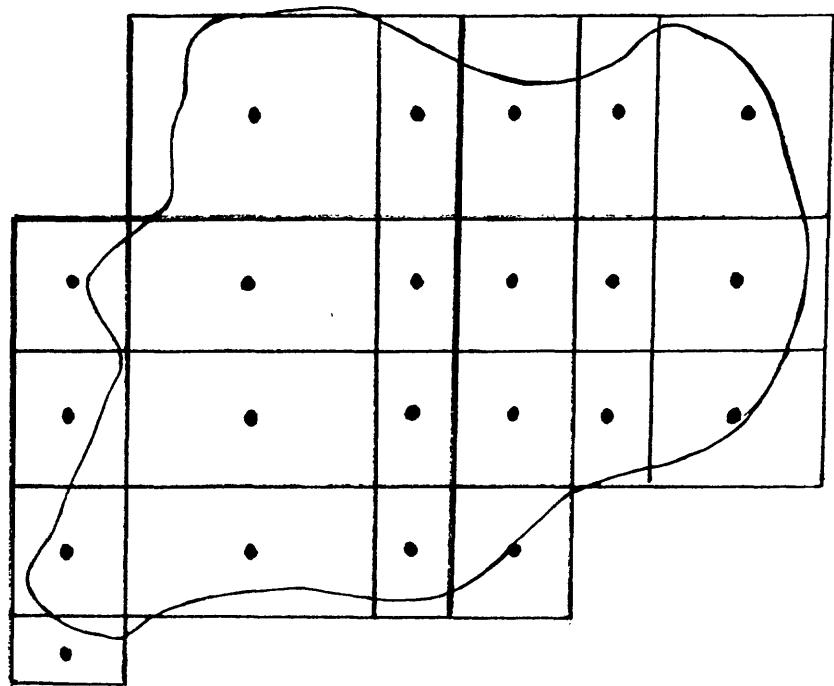


Figure 2. Finite-difference grid showing the locations of the reservoir, the grid blocks and nodes.

## Finite-Difference Representation

Equations 1 and 2 may be written in compact, implicit finite-difference form as:

$$\Delta_x [(T_{wx} + T_{sx}) \Delta_x p^{n+1}] + \Delta_y [(T_{wy} + T_{sy}) \Delta_y p^{n+1}] \\ + v q_m' = \frac{1}{\Delta t} (M^{n+1} - M^n) , \quad (22)$$

and

$$\Delta_x (T_{hx} \Delta_x p^{n+1}) + \Delta_y (T_{hy} \Delta_y p^{n+1}) + \Delta_x (T_{cx} \Delta_x h^{n+1}) \\ + \Delta_y (T_{cy} \Delta_y h^{n+1}) + v q_h'' = \frac{1}{\Delta t} (E^{n+1} - E^n) , \quad (23)$$

where the right sides of equations 22 and 23 are the accumulation terms for mass and energy, respectively, and  $q_m'$  and  $q_h''$  are the total mass and heat lost to wells, respectively.

The transmissibility terms  $T_w$ ,  $T_s$ ,  $T_h$ , and  $T_c$ , are given by:

$$T_w = (kA/\ell) \rho_w k_{rw} / \mu_w , \quad (24a)$$

$$T_s = (kA/\ell) \rho_s k_{rs} / \mu_s , \quad (24b)$$

$$T_h = T_w h_w + T_s h_s + (AK_m / \ell) (\frac{\partial T}{\partial p})_h , \quad (24c)$$

$$T_c = (AK_m / \ell) (\frac{\partial T}{\partial h})_p , \quad (24d)$$

and the mass and energy terms  $M$  and  $E$  are:

$$M = V\phi\rho , \quad (25a)$$

$$E = V[\phi\rho h + (1-\phi)\rho_r h_r] , \quad (25b)$$

where  $V$ ,  $A$  and  $\ell$  are the grid block volume, cross-sectional area perpendicular to the flow direction, and the length increment in the flow direction, respectively. The difference operator acts as follows:

$$\begin{aligned} \Delta_x (T_{wx} \Delta_x p^{t+\Delta t}) &= T_{wx_{j+\frac{1}{2},j}} (p_{i+1,j}^{t+\Delta t} - p_{i,j}^{t+\Delta t}) \\ &\quad - T_{wx_{i-\frac{1}{2},j}} (p_{i,j}^{t+\Delta t} - p_{i-1,j}^{t+\Delta t}) , \end{aligned} \quad (26)$$

where  $i$  and  $j$  are indices in the  $x$ - and  $y$ - directions, and  $t$  is the index for the time level.

The interblock transmissibility terms (values at  $i + \frac{1}{2}$ ,  $i - \frac{1}{2}$ ,  $j + \frac{1}{2}$  and  $j - \frac{1}{2}$ ) are composed of two parts: that which is a function of space only (for example,  $kA/\ell$ ) and that which is a nonlinear function of pressure and/or enthalpy (for example  $\rho k_p/\mu$ ). To approximate these terms requires averaging or weighting of the various components over each grid block. For the space dependent part, this is accomplished by using a harmonic mean, for example,

$$(kA/\ell)_{i+\frac{1}{2}} = \frac{2k_{i+1}k_i A_{i+1} A_i}{k_{i+1} A_{i+1} \ell_i + k_i A_i \ell_{i+1}} \quad (27)$$

The nonlinear part of the transmissibility terms are generally assigned the upstream value. The upstream node is determined by comparing the pressures at ( $i$ ) and ( $i+1$ ), and using the larger pressure to compute the nonlinear part. Alternatively this part may be determined by a length weighted arithmetic average, for example,

$$k_{rw}_{i+\frac{1}{2}} = \frac{k_{rw_{i+1}} \ell_{i+1} + k_{rw_i} \ell_i}{\ell_{i+1} + \ell_i} \quad (28)$$

Of the two procedures, upstream weighting yields a lower order approximation of the spatial derivative but exhibits a more stable solution.

## Solution Procedure

The difference equations 22 and 23 are solved simultaneously for the unknown pressure and enthalpy in each grid block for each time step. Since equations 22 and 23 are nonlinear, a provision is included to iterate on nonlinear coefficients. Newton-Raphson iteration is used on the accumulation terms and Picard iteration is used on the coefficients of the spatial derivatives. Two difference equations are obtained for each grid block, and the resulting system of  $2n$  equation has the form,

$$[B] \{X^{t+\Delta t}\} - \{f(X^{t+\Delta t})\} + \{f(X^t)\} + \{q\} = 0 , \quad (29)$$

where the superscript indicates the time level, the matrix  $[B]$  incorporates the transmissibility terms, the vector  $\{X\}$  contains the unknown pressure and enthalpy values, that is

$$\{X\} = \begin{Bmatrix} p_1 \\ h_1 \\ p_2 \\ h_2 \\ \vdots \\ \vdots \\ p_n \\ h_n \end{Bmatrix}$$

and the vector  $\{f(X)\}$  is a non-linear function describing the accumulation terms.

To linearize 29, an iterative technique called the Newton-Raphson procedure is applied. The iteration level is indicated by a subscript, and in particular the first iteration (initial guess) is indicated by the subscript (0). Substitution of the initial guess into equation 29 yields a residual:

$$\begin{aligned} \left[ B_0 \right] \{x_0^{t+\Delta t}\} - \{f(x_0^{t+\Delta t})\} + \{f(x^t)\} \\ + \{q_0\} = \{R(x_0^{t+\Delta t})\}. \end{aligned} \quad (30)$$

For the first time step, the initial conditions are used as the initial guess; for subsequent time steps, the results from the previous time step are used as the initial guess.

If the residual is expanded in a truncated first order Taylor series, one obtains,

$$\{R(x^{t+\Delta t})\} = \{R(x_0^{t+\Delta t})\} + \{\Delta X\} \left[ \frac{\partial \{R(x_0^{t+\Delta t})\}}{\partial x_0^{t+\Delta t}} \right] \doteq 0 ,$$

or rearranging,

$$\left[ \frac{\partial \{R(x_0^{t+\Delta t})\}}{\partial x_0^{t+\Delta t}} \right] \{\Delta X\} = - \{R(x_0^{t+\Delta t})\}. \quad (31)$$

Taking the derivatives with respect to the unknown vector  $\{X\}$  equation 35 yields,

$$\left[ \frac{\partial \{R(x_0^{t+\Delta t})\}}{\partial x_0^{t+\Delta t}} \right] = \left[ B_0 \right] - \left[ \frac{\partial \{f(x_0^{t+\Delta t})\}}{\partial x_0^{t+\Delta t}} \right] , \quad (32)$$

where  $\left[ \frac{\partial \{f(x_0^{t+\Delta t})\}}{\partial x_0^{t+\Delta t}} \right]$  is obtained by taking the partial derivatives of the accumulation terms with respect to pressure and enthalpy. Specifically, by use of the chain rule:

$$b \frac{\partial(\phi\rho)}{\partial p} = b \left[ \rho \frac{d\phi}{dp} + (\frac{\partial \rho}{\partial p})_h \right]$$

$$b \frac{\partial(\phi\rho)}{\partial h} = b(\frac{\partial \rho}{\partial h})_p ,$$

$$\begin{aligned} b \frac{\partial}{\partial p} \left[ \phi\rho h + (1-\phi)\rho_r h_r \right] &= b \left[ (\rho h - \rho_r h_r) \frac{d\phi}{dp} \right. \\ &\quad \left. + h(\frac{\partial \rho}{\partial p})_h + (1-\phi)\rho_r c_r (\frac{\partial T}{\partial p})_h \right] , \end{aligned}$$

and

$$b \frac{\partial}{\partial h} \left[ \phi\rho h + (1-\phi)\rho_r h_r \right] = b \left[ \phi h(\frac{\partial \rho}{\partial h})_p + (1-\phi)\rho_r c_r (\frac{\partial T}{\partial h})_p + \phi\rho \right] .$$

Substituting 32 into equation 31 gives,

$$\left[ \left[ B_0 \right] - \left[ \frac{\partial \{f(x_0^{t+\Delta t})\}}{\partial x_0^{t+\Delta t}} \right] \right] \{ \Delta X \} = - \{ R(x_0^{t+\Delta t}) \} , \quad (33)$$

which is the matrix equation to be solved.

The new values of  $x^{t+\Delta t}$  are determined from,

$$\{x_i^{t+\Delta t}\} = \{x_{i-1}^{t+\Delta t}\} + \{\Delta x\}, \quad (34)$$

where  $i = 1, 2, \dots, m$ ,  $m$  being the specified number of iterations. If  $m = 1$ , this procedure is equivalent to the residual formulation outlined by Weinstein, Stone, and Kwan (1969).

In most of the two-dimensional, areal simulations studied it was not necessary to use Newton-Raphson iteration on the nonlinear coefficients in  $[B]$  and  $\{q\}$  to obtain satisfactory solutions. For radial problems and vertical cross-sections this is not the case as pointed out by Toronyi and Farouq Ali (1975). As stated earlier, Picard iteration is used for  $[B]$  and  $\{q\}$ . In this process the nonlinear coefficients are simply updated on each iteration.

The solution of the linearized matrix equation 33 is solved using the Gauss-Doolittle method for banded, nonsymmetric matrices. The coefficient matrix is decomposed into the product of upper and lower triangular matrices, from which the solution may be determined by back substitution. Because the coefficient matrix is nonlinear, this solution procedure is required for each iteration of each time step. The general form of the coefficient matrix and the form in which it is stored in the program is shown in figure 3, where MDIM is the matrix bandwidth and NDIM is the number of rows.

MDIM				
(1,1),	(1,2),	0 ,	0 ,	0
(2,1),	(2,2),	(2,3),	0 ,	0
0 ,	(3,2),	(3,3),	(3,4),	0
0 ,	0 ,	(4,3),	(4,4),	(4,5)
0 ,	0 ,	0 ,	(5,4),	(5,5)

- a. Normal storage of matrix where bold marks indicate the main diagonal.

0 ,	(1,1),	(1,2)
(2,1),	(2,2),	(2,3)
(3,2),	(3,3),	(3,4)
(4,3),	(4,4),	(4,5)
(5,4),	(5,5),	0

- b. Banded storage of matrix showing new location of the main diagonal.

Figure 3. Normal storage of matrix and banded storage of matrix.

## Heat Loss Calculations

Geothermal reservoirs occur in areas of anomalously high heat flow. The effects of the high geothermal gradient and the loss and gain of heat to the base and cap rock are important factors that are incorporated into the reservoir model. In a typical hydrothermal system, heat flows into the reservoir at the base and out at the top.

In order to obtain the heat flux through the confining beds three simplifying assumptions are made. First, the permeability in the confining bed is assumed to be low and convective flow may be neglected; thus, only the heat conduction equation needs to be solved. Further, the horizontal conduction terms in the confining beds are assumed to be small relative to the vertical terms, and may be neglected. For petroleum reservoirs undergoing thermal recovery, it has been demonstrated that the effect of horizontal conduction in the confining beds is small (Chase and O'Dell, 1973). This assumption leads to the one-dimensional, heat-conduction equation:

$$K_r \frac{\partial^2 T'}{\partial z^2} = (\rho c)_r \frac{\partial T'}{\partial t} ,$$

where the vertical thermal conductivity, density, and heat capacity of the confining bed are considered constant. The final assumption is that the geothermal reservoir, prior to exploitation, is at steady state; that is, the heat entering at the base equals that leaving at the top and the net heat gain of the reservoir is zero. This assumption allows consideration of only the "transient" heat flow caused by temporal temperature changes in the reservoir.

Therefore, the one-dimensional, conduction equation may be reformulated as follows:

$$k_r \frac{\partial^2 T^*}{\partial z^2} = (\rho c)_r \frac{\partial T^*}{\partial t} , \quad (35)$$

where  $T^* = T' - T'_0$  ( $T'$  is the temperature in the confining bed and  $T'_0$  is the initial temperature in the confining bed.)

Equation 35 is subject to initial and boundary conditions. The initial conditions are simply  $T^*(z,0) = 0$ . For the boundary condition at the top of the cap rock, the temperature change is assumed to be zero for all time. At the cap rock-reservoir boundary, a step function in temperature is used. It is determined using the difference between the initial reservoir temperature and the reservoir temperature at the last time step (that is, the step function is lagged by one time step). Once the  $T^*$  distribution through the cap rock is determined, Fourier's equation is used to compute the heat flux.

A similar set of boundary and initial conditions apply to the base rock; however, instead of solving equation 35 for both the top and bottom, it is assumed that the two fluxes are approximately equal and therefore the total heat leakage is obtained by multiplying the flux computed at the top by two. This assumption is valid since only the "transient" heat leakage is considered; however, this portion of the program could be modified if needed (for example, if steady state thermal gradients above and below the reservoir are significantly different).

In the numerical model, equation 35 is actually solved at each grid block for each iteration. It is solved using a Galerkin, finite-element approximation for the space derivative combined with an implicit difference approximation for the time derivative. Linear elements are used with a variable mesh generator that divides the confining bed thickness into ten elements that double in size with distance from the reservoir boundary. Therefore, the element adjacent to the reservoir is relatively small, and it is the temperature difference across this element that is used in Fourier's equation to compute the heat flux.

## MODEL DOCUMENTATION

### Notes on Use of Program

- (1) This version of the finite-difference model is restricted to problems involving confined, horizontal reservoirs exhibiting two-dimensional flow. Furthermore, the reservoir is overlain and underlain by impermeable layers that allow only conduction of heat.
- (2) To minimize core requirements, the dimensions of the arrays A(NBB,MBE) and R(NBB) must be specified for each problem, where

$$NBB = 2 * NB$$

(NB - number of nonzero blocks, that is, blocks that have nonzero permeability) and MBE is the estimated matrix bandwidth given for two simultaneous unknowns at each grid block by

$$MBE = 2 * (2 * MM + 1) + 1$$

where,

$$MM = NY \text{ IF } NX > NY$$

$$MM = NX \text{ IF } NY > NX$$

and NX and NY are the number of columns and rows, respectively.

- (3) The actual matrix bandwidth is computed internally and printed. If it differs from the estimated bandwidth, change MBE so that it is equal to the actual matrix bandwidth. Also, change the dimensions of array A.
- (4) The regression equations used in this program are based on steam table data for a temperature range of 10° to 300°C.
- (5) The units of the input data must be in the cgs system.
- (6) At present most arrays are dimensioned to solve problems with a maximum of 20 columns and 10 rows.

- (7) Relative permeability functions described in equations 10 and 11 are programmed in SUBROUTINE PRPTY statement numbers PRP1230 - PRP1300 where the residual water saturation,  $S_{wr} = 0.3$  and the residual steam saturation,  $S_{sr} = 0.05$ . Other equations may be substituted for these; the only restriction is that relative permeability must be a smooth function of saturation. A non-smooth relationship (such as linear interpolation between data points) can result in an oscillatory, unstable solution.
- (8) The two-dimensional treatment in this model assumes that the fluid properties are uniform with depth. This assumption is probably valid only for very thin reservoirs, but may be a suitable approximation for some applications.
- (9) The program should be in double precision, except when using a computer having single precision accuracy to 10 significant digits, in which case remove the REAL\*8 IMPLICIT cards.
- (10) Although the unknown dependent variables are pressure and enthalpy, the user is given the option to read in either initial pressures and temperatures ( $KOD9 = 1$ ) or initial pressures and enthalpies ( $KOD9 = 0$ ). This option is provided since field temperatures are more readily available than enthalpies. If temperatures are read, they are converted in the program to enthalpies, and subsequent calculations are made using the enthalpy values. If the initial conditions of the reservoir are two phase, the user must read in enthalpies.

- (11) To reduce the number of lines in the output, only the computed pressures and enthalpies are printed for each time step. The user may, however, also have the computed water saturations, temperatures, and fluid densities printed as often as desired by specifying the proper value for IPRT. This parameter allows the additional data to be printed every IPRT time step. For example, if IPRT = 10, saturations, temperatures and densities will be printed on time step 1, 10, 20, . . . until the end of the simulation.
- (12) Minimum number of blocks required for a successful run are two in the x-direction and two in the y-direction.

## Description of the Subroutines

The FØRTRAN IV code contains a main program and 12 subroutines, which are shown diagrammatically in Figure 4. The purpose of each subroutine is listed below.

MAIN Driving program for the subroutines. In addition, the large arrays, A and R, are dimensioned to minimize core requirements.

GDATA Reads and writes problem information according to the formats listed in the INPUT section of this report.

READ Reads the two dimensional arrays containing data for each finite-difference block.

TCALC Computes the interblock transmissibility terms. Intrinsic permeability is determined as a harmonic mean of the values in the two blocks.

MATR Computes and prints matrix bandwidth. The estimated bandwidth should equal the computed bandwidth. The bandwidth is used in dimensioning arrays, and if the estimated and computed bandwidths are not equal, computational errors could result. See the section on 'Notes on Use of Program' for details on how to calculate the estimated bandwidth.

PRPTY Computes thermodynamic properties based on regression equations determined using data from steam tables.

VERTCD Computes vertical conductive heat leakage through a confining bed. This is accomplished by solving the one-dimensional, heat-conduction equation at each finite-difference grid block. The numerical method used in this subroutine is the finite-element method.

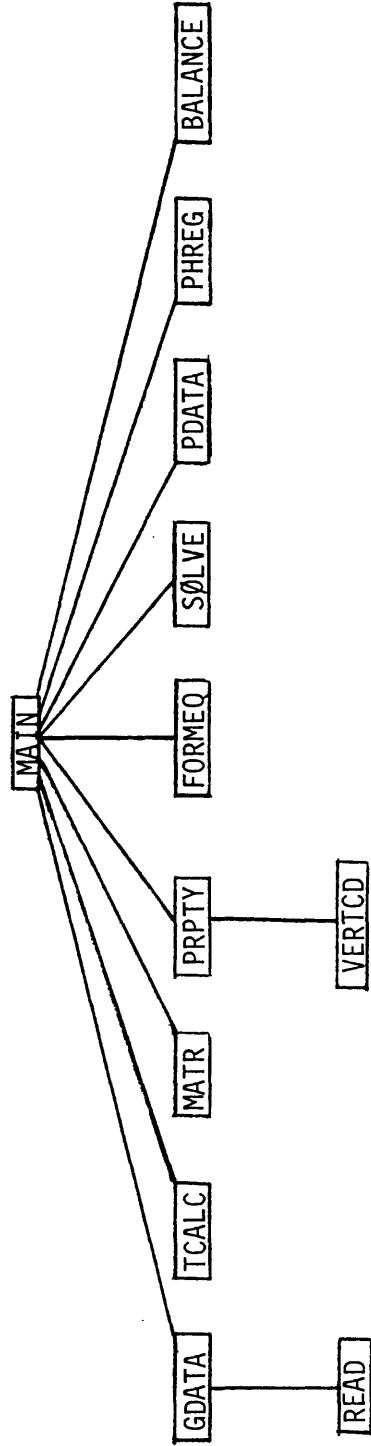


Figure 4. Program subroutines showing order and link of calling.

FØRMEQ Forms the final matrix equation. Note that the matrix is nonsymmetric and that the dependent variables, pressure and enthalpy are solved simultaneously.

SØLVE Solves the matrix equation using the Gauss-Doolittle method. It triangularizes a banded nonsymmetric matrix and then back substitutes.

PDATA Prints the computed pressures and enthalpies for each time step.

PHREG Determines the thermodynamic region for each finite-difference grid block.

BALNCE Computes a mass and energy balance for each time step.

A generalized flow chart showing the approximate order that the subroutines are used is shown in Figure 5.

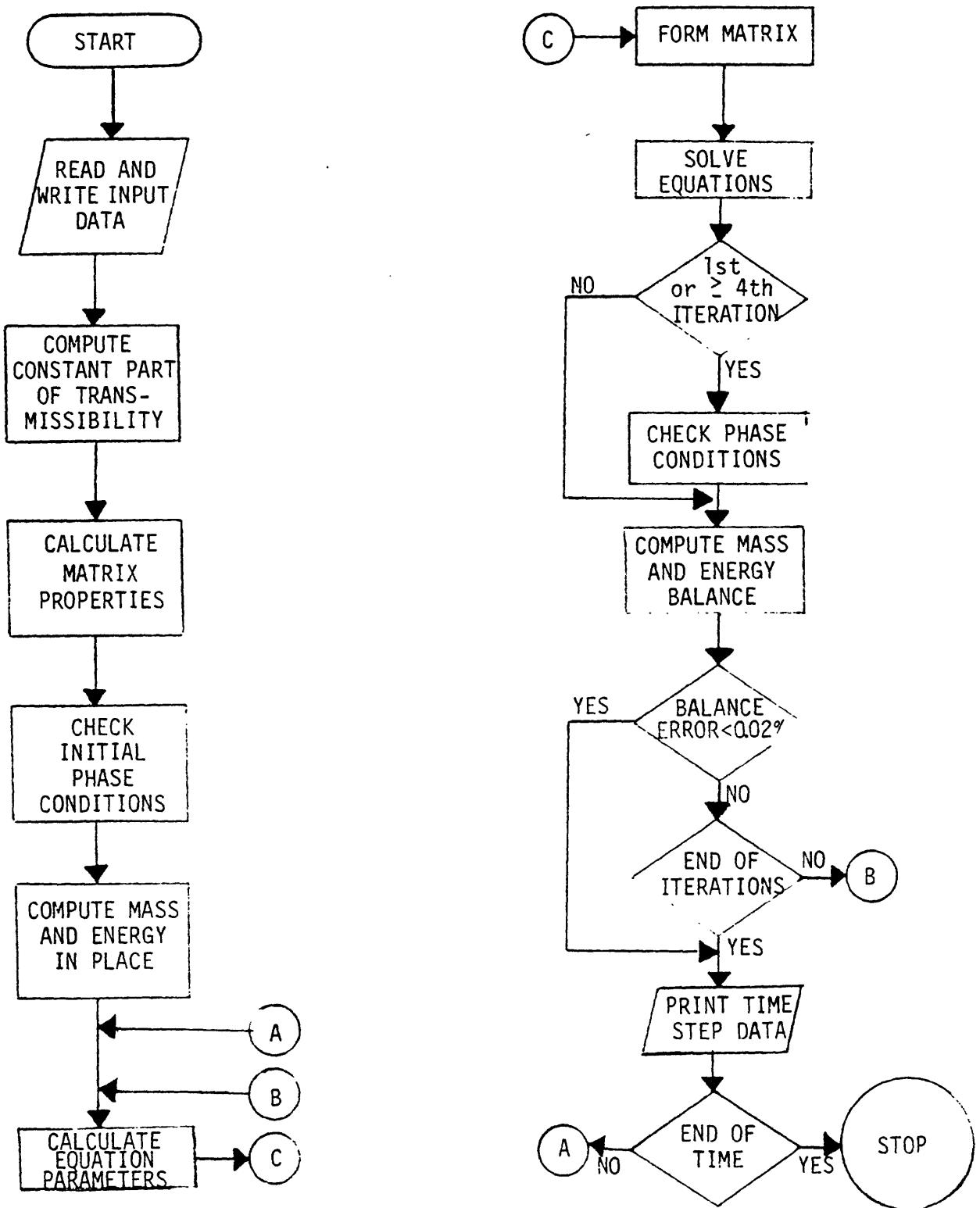


Figure 5. Generalized flow chart.

## Input

<u>Columns</u>	<u>Format</u>	<u>Name</u>	<u>Description</u>
<i>Card 1</i>			
1-80	20A4	TITLE	Problem title
<i>Card 2</i>			
1-5	I5	NX	Number of columns (x-direction)
6-10	I5	NY	Number of rows (y-direction)
11-15	I5	NB	Number of non-zero blocks (those with non-zero permeability)
16-20	I5	NK	Maximum number of Newton iterations
21-25	I5	NT	Maximum number of time steps
26-30	I5	NS	Number of sources
31-35	I5	MBE	Estimated bandwidth (see text)
36-40	I5	I0PT	Read 1 for upstream weighting; 2 for midpoint weighting (See section on Finite-Difference Representation)
41-45	I5	PPRT	Number of time steps between printing thermodynamic data
<i>Card 3</i>			
1-10	G10.0	DELT	Time step (in seconds)

<u>Columns</u>	<u>Format</u>	<u>Name</u>	<u>Description</u>
<i>Card 4</i>			
1-5	I5	KØD1	Read 1 if x-spacing is constant (otherwise, leave blank)
6-10	I5	KØD2	Read 1 if y-spacing is constant
11-15	I5	KØD3	Read 1 if initial pressure is constant
16-20	I5	KØD4	Read 1 if initial enthalpy is constant
21-25	I5	KØD5	Read 1 if x-permeability is constant
26-30	I5	KØD6	Read 1 if y-permeability is constant
31-35	I5	KØD7	Read 1 if initial porosity is constant
36-40	I5	KØD8	Read 1 if reservoir thickness is constant
41-45	I5	KØD9	Read 1 if temperature is read in place of enthalpy
<i>Data Set 1 - X-spacing</i>			
1-80	8G10.0	DX(I)	Spacing in the x-direction (NX values); if constant, KØD1 = 1 and only read one value (cm)
<i>Data Set 2 - Y-spacing</i>			
1-80	8G10.0	DY(J)	Spacing in the y-direction (NY values); if constant, KØD2 = 1 and only read one value (cm)
<i>Data Set 3 - Initial pressure*</i>			
1-80	8G10.0	P(I,J)	Initial pressure distribution in the reservoir; if constant, KØD3 = 1 and only read one value (dynes/cm <sup>2</sup> )

\*Start new card for the beginning of each new row (may start with the top row or bottom row, but be consistent) leaving blanks for missing blocks.

<u>Columns</u>	<u>Format</u>	<u>Name</u>	<u>Description</u>
<i>Data Set 4 - Initial enthalpy *†</i>			
1-80	8G10.0	H(I,J)	Initial enthalpy distribution in the reservoir; if constant, KØD4 = 1 and only read one value (ergs/g)
<i>Data Set 5 - X-permeability*</i>			
1-80	8G10.0	XK(I,J)	Reservoir permeability in x-direction; if constant, KØD5 = 1 and only read one value ( $\text{cm}^2$ )
<i>Data Set 6 - Y-permeability*</i>			
1-80	8G10.0	YK(I,J)	Reservoir permeability in y-direction; if constant, KØD6 = 1 and only read one value ( $\text{cm}^2$ )
<i>Data Set 7 - Porosity*</i>			
1-80	8G10.0	PHI(I,J)	Reservoir porosity; if constant, KØD7 = 1 and only read one value (dimensionless)
<i>Data Set 8 - Thickness*</i>			
1-80	8G10.0	DZ(I,J)	Reservoir thickness; if constant, KØD8 = 1 and only read one value (cm)

\* Start new card for the beginning of each new row (may start with the top row or bottom row, but be consistent) leaving blanks for missing blocks.

† If KØD9 = 1, read initial temperature distribution ( $^{\circ}\text{C}$ ) instead.

<u>Columns</u>	<u>Format</u>	<u>Name</u>	<u>Description</u>
<i>Data Set 9 - Sources</i>			
1-5	I5	I	Column number of well
6-10	I5	J	Row number of well
11-25	G15.0	Q(I,J)	Strength (g/s) of source/sink at block i,j

Note: 1-NS cards; if NS = 0 this data set is omitted.

*Card 5*

1-10	G10.0	XKC	Medium thermal conductivity of the reservoir (ergs/s-cm°C)
11-20	G10.0	COND	Confining bed thermal conductivity (ergs/s-cm°C)
21-30	G10.0	PHFWT	Rock enthalpy derivative with respect to temperature (specific heat) (ergs/g°C)
31-40	G10.0	DF	Rock density (g/cm <sup>3</sup> )
41-50	G10.0	BETA	Compressibility of reservoir (cm <sup>2</sup> /dyne)

## Output

To aid the user in detecting errors associated with data input, data that is read in is immediately printed; thus output appears in the following order:

Title of Problem

Finite-Difference Data

Grid Numbers

Time Parameters

Codes

Spacing in X-Direction

Spacing in Y-Direction

Initial Pressure

Initial Enthalpy

X-Permeability

Y-Permeability

Initial Porosity

Reservoir Thickness

Sources

Rock Properties

Maximum Bandwidth

In addition, every IPRT time step the following is also printed:

Water Saturations  
Temperatures  
Density

}

Printed at beginning of time step,  
but are based on pressure and enthalpy  
from previous time step.

**Finally, on a successful run, the following is printed every time step:**

**Step Number**

**Time**

**Pressure Values**

**Enthalpy Values**

**Mass and Energy Balance**

## APPLICATIONS

### Example 1

The linear flow of hot incompressible fluid through a confined aquifer may be described by the following equations:

$$K_r \frac{\partial^2 u}{\partial z^2} = \rho_r c_r \frac{\partial u}{\partial t}, \quad z > 0, \quad t > 0 \quad (36a)$$

$$K_t \frac{\partial^2 u}{\partial x^2} - v_w \rho_w c_w \frac{\partial u}{\partial x} + \frac{2}{b} K_r \frac{\partial u}{\partial z} = \rho_t c_t \frac{\partial u}{\partial t}, \quad z = 0, \quad x > 0, \quad t > 0 \quad (36b)$$

subject to:

$$u(x,0) = 0, \quad x > 0$$

$$u(0,t) = 1, \quad z = 0, \quad t \geq 0$$

$$\lim_{x^2 + z^2 \rightarrow \infty} u = 0$$

$$x^2 + z^2 \rightarrow \infty$$

where  $u = \frac{T-T_0}{T_1-T_0}$  is the normalized temperature;  $T_0$  is the overburden (underburden) temperature and the initial aquifer temperature; and  $T_1$  is the temperature of the injection fluid. The subscripts refer to the rock, r, water, w, and total (rock and water), t.

Avdonin (1964) presents an analytical solution for equation (36):

$$u(x, \tau) = \frac{x}{\sqrt{\pi\tau}} \int_0^1 \left\{ \exp \left[ - (s\gamma\sqrt{\tau} - \frac{x}{2s\sqrt{\tau}})^2 \right] \operatorname{erfc} \left( \frac{\alpha s^2 \sqrt{\tau}}{2\sqrt{1-s^2}} \right) \right\} \frac{ds}{s^2} \dots$$

$$\text{where } x = \frac{2x}{b}; \quad \tau = \frac{4K_t t}{c_t \rho_t b^2}; \quad \gamma = \frac{Q c_w \rho_w}{4K_t};$$

$$\alpha = \sqrt{\frac{K_r c_r \rho_r}{K_t c_t \rho_t}} \quad ; \text{ and } Q \text{ is the injection flow rate. (In the}$$

original reference,  $\gamma$  is defined with an 8 in the denominator; this appears to be a typographical error.)

To simulate this problem, the following assumptions were made:

1) Temperature is a function of enthalpy only, according to:

$$T = - 0.0208 + 2.39 \times 10^{-8} h \quad (38)$$

2) Density is a function of pressure only, according to:

$$\rho = 0.989875 + 4.00894 \times 10^{11} p \quad (39)$$

3) Porosity is constant.

These changes must be made in the program by appropriately changing the single-phase (water) statement functions for temperature and density, and by appropriately changing their respective pressure- and enthalpy-derivatives. For constant porosity, beta is read as zero. Other parameters and the initial conditions used in this example are given in table 1. A block-centered grid consisting of 20 blocks was used, and the time step was  $6.25 \times 10^6$  sec. The total simulation time was  $3.75 \times 10^8$  sec.

The results are presented in figure 6. This problem was chosen because it exhibits pronounced truncation error in the approximation of the spatial derivatives. Time steps were chosen to reduce time truncation errors. Both mid-point weighting and upstream weighting were used. It is clear that mid-point weighting approximates the temperature front better than upstream weighting, but exhibits oscillations at the base of the front. Upstream weighting smears the front out by numerical diffusion and does not exhibit oscillations.

TABLE 1 - DATA FOR AVDONIN LINEAR EXAMPLE

PARAMETER	SYMBOL	VALUE
velocity	v	$1.28 \times 10^4$ cm/sec
porosity	$\phi$	0.20
reservoir thermal conductivity	$K_m$	$3.20 \times 10^6$ ergs/sec-cm $^{\circ}$ C
confining bed thermal conductivity	$K'$	$3.20 \times 10^5$ ergs/sec-cm $^{\circ}$ C
rock density	$\rho_r$	$2.50$ g/cm $^3$
rock specific heat	$c_r$	$1.01 \times 10^7$ ergs/g $^{\circ}$ C
aquifer thickness	b	$2.00 \times 10^4$ cm
initial pressure	$p_i$	$1.38 \times 10^7$ dynes/cm $^2$
initial enthalpy	$h_i$	$3.35 \times 10^9$ ergs/g
initial fluid viscosity	$\mu_i$	$3.58 \times 10^{-3}$ g/cm-sec
initial temperature	$T_i$	$80.19$ $^{\circ}$ C
injection temperature	$T'$	$40.01$ $^{\circ}$ C
fluid density	$\rho$	$0.99$ g/cm $^3$
fluid specific heat	$c_w$	$4.18 \times 10^7$ ergs/g $^{\circ}$ C

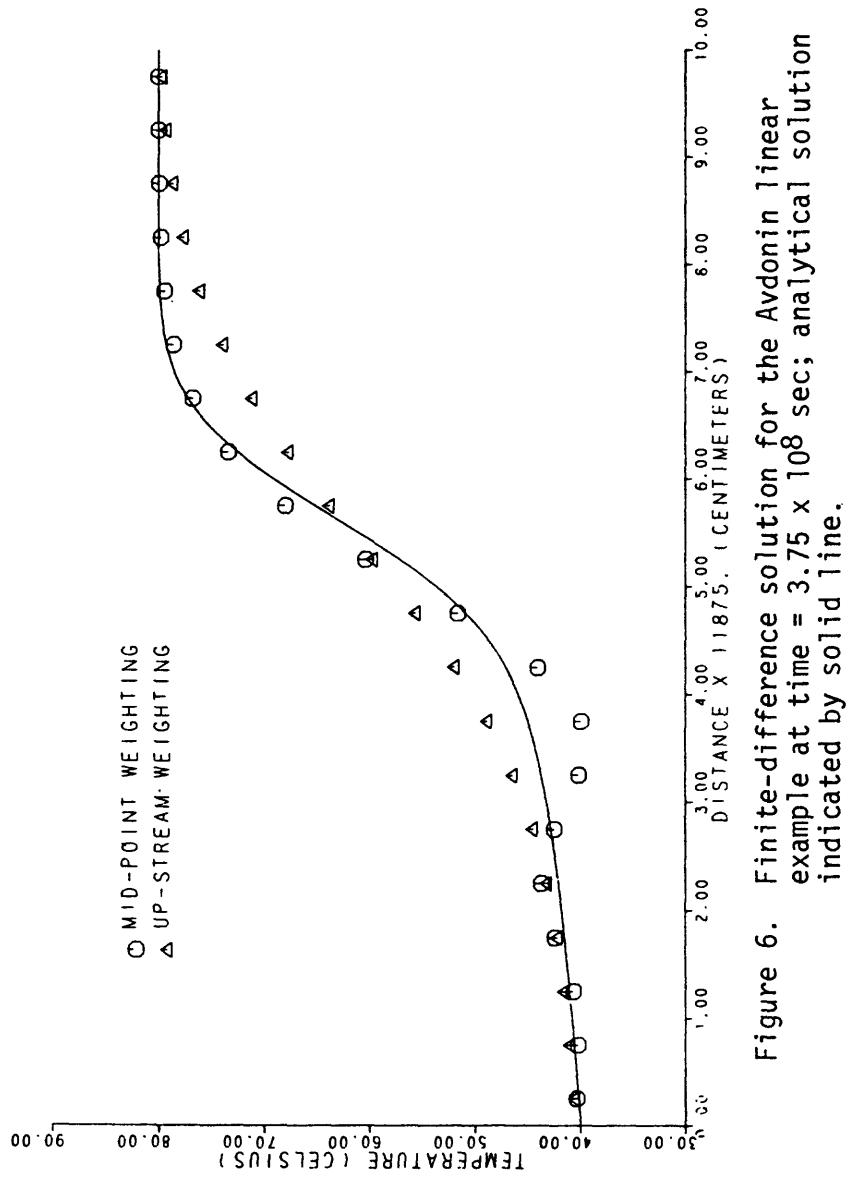


Figure 6. Finite-difference solution for the Avdonin linear example at time =  $3.75 \times 10^8$  sec; analytical solution indicated by solid line.

## Example 2

This example was designed to demonstrate the behavior of an initially hot-water geothermal reservoir that develops a two-phase zone under the influence of production. This problem was originally solved using a finite-element model and a five year simulation period (Faust and Mercer, 1975) and was rerun later under the same conditions using a finite-difference model (Faust and Mercer, 1976). The finite-difference simulation in this report is the same as that presented in the above references, however this simulation stops after 104 days.

Data for this example is given in table 2. This is a one-dimensional problem and therefore only one row is used in conjunction with ten columns, giving ten non-zero blocks. It is assumed that the reservoir is closed to the flow of heat and mass at all boundaries. The time step is  $5.0 \times 10^5$  seconds and the maximum number of time steps is 18, with a maximum of twelve iterations. The estimated bandwidth is seven, and the spacings in the x- and y-directions are  $2.0 \times 10^4$  cm and  $5.0 \times 10^4$  cm respectively. Upstream weighting is used and there is only one sink located (at the left end) in the first block (1,1). A listing of the formated model data is given in figure 7. Finally, the relative permeabilities used for this problem are:

$$k_{rw} = [(S_w - 0.35)/0.65]^4 \quad (40)$$

and

$$k_{rs} = \{1.0 - [(S_w - 0.35)/0.65]^2\} * \{1.0 - [(S_w - 0.35)/0.65]\}^2 . \quad (41)$$

TABLE 2 - DATA FOR EXAMPLE WITH CONVERSION

porosity	$\phi$	0.10
reservoir thermal conductivity	$K_m$	$3.20 \times 10^5$ ergs/sec-cm°C
rock density	$\rho_r$	$2.50$ g/cm <sup>3</sup>
rock specific heat	$c_r$	$1.01 \times 10^7$ ergs/g°C
aquifer thickness	$b$	$5.00 \times 10^4$ cm
initial pressure	$p_i$	$4.38 \times 10^7$ dynes/cm <sup>2</sup>
initial enthalpy	$h_i$	$1.02 \times 10^{10}$ ergs/g
initial temperature	$T_i$	$236.12^\circ\text{C}$
initial density	$\rho_i$	$0.82$ g/cm <sup>3</sup>
permeability	$k$	$1.00 \times 10^{-10}$ cm <sup>2</sup>
discharge rate	$Q$	$2.00 \times 10^4$ g/sec

Figure 7. Formated Data for Example 2.

EXAMPLE RUN WITH CONVERSION

10	1	10	12	18	1	7	1	1
500000.								
1	1	1	1	1	1	1	1	
20000.								
50000.								
.438D8								
1.024D10								
0.1D-9								
0.1D-9								
0.1								
50000.								
1	1	-20000.						
3.2D5		0.00000						
			1.0107		2.5			
							0.000	

These relationships may be found in the program at statement numbers PRP1230 - PRP1300. For problems with different functions, the user must change these statements. Note that experience has shown that these functions must be smooth and continuous or numerical oscillations may result.

At the end of 17 time steps ( $8.5 \times 10^6$  sec) the finite-difference model predicts that the water saturation in the first block is 0.96994; all other blocks are at a saturation of 1.0. For illustrative purposes results from the finite-element and finite-difference models (Faust and Mercer, 1976) for the five year simulation are compared in figures 8 and 9. In figure 8, pressure is plotted versus distance (origin is at the left end of the reservoir), for various simulation times. The specified times represent the duration of exploitation. In this example, the pressure drops rapidly in the early stages of production, and at the end of one month the first block has become two phase. When this occurs, the pressure continues to drop rapidly in the rest of the reservoir, but changes only slightly in the block containing the sink. This is expected, because once a block becomes two-phase, the pressure is maintained by the formation of steam. Also, note that once the system becomes two phase, the time step may be increased without causing any numerical problems. After 3 years of exploitation, the pressures in the entire reservoir have lowered to a point just slightly above the saturated-vapor pressure. Subsequent mass extraction results in reduced water saturations in elements near the sink, and pressures drop very slowly.

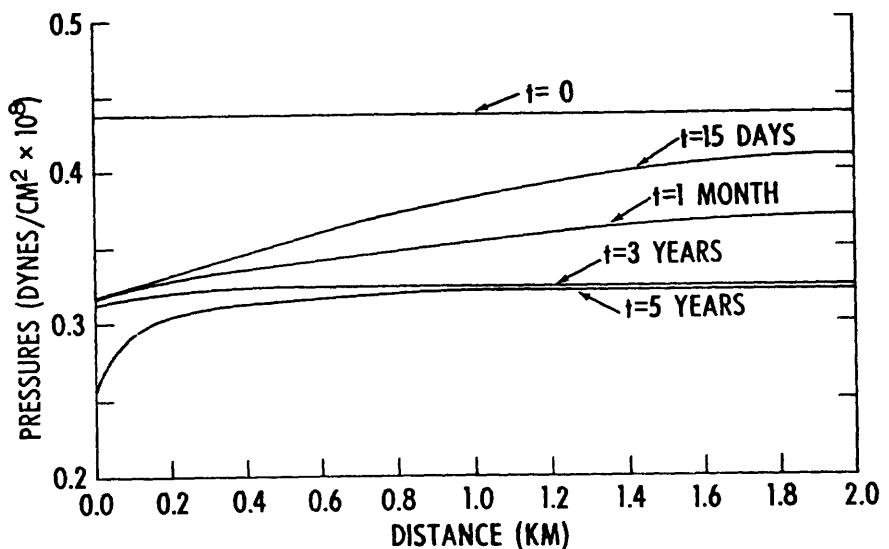


Figure 8.- Computed pressure distribution at various times for example 2. (After Faust and Mercer, 1976.)

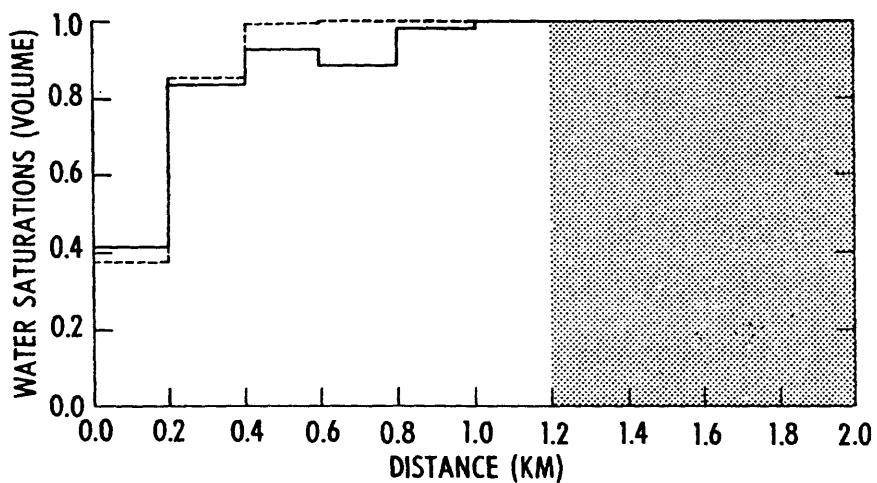


Figure 9. Computed water saturation distribution for example 2 after 5 years of exploitation.  
 (Single-phase water indicated by shading;  
 solid line: Galerkin finite-element solution;  
 dashed line: finite-difference solution.)  
 After Faust and Mercer, 1976.

Water saturation distributions at the end of 5 years of exploitation predicted by both the Galerkin- and finite-difference models are shown in figure 9. Note that the finite-element solution exhibits an oscillation in the saturation distribution. The upstream-weighting scheme used in the finite-difference model eliminates this oscillation.

## NOTATION

Parameter	Coded Name	Description
b	DZ	reservoir thickness
c <sub>r</sub>	PHFWT	rock specific heat
h	H	enthalpy
h <sub>r</sub>	HROCK	rock enthalpy
h <sub>s</sub>	HS	steam enthalpy
h <sub>w</sub>	HW	water enthalpy
K <sub>m</sub>	XKC	medium thermal conductivity
K <sub>r</sub>	CØND	confining bed thermal conductivity
k	XK	x- permeability
	YK	y- permeability
k <sub>r</sub>	XKW	water relative permeability
	XKS	steam relative permeability
p	P	pressure
q <sub>m</sub> *	Q	mass source term
q <sub>h</sub> *	QH	energy source term
q <sup>11</sup>	CQ	vertical conductive energy source term
S <sub>w</sub>	SW1	water volume saturation
T	TEMP2	temperature
t	TIME	simulation time
β	BETA	rock compressibility
μ <sub>s</sub>	VS	steam viscosity

Parameter	Coded Name	Description
$\mu_w$	VW	water viscosity
$\rho$	DEN2	density
$\rho_r$	DF	rock density
$\rho_s$	DS	steam density
$\rho_w$	DW	water density
$\phi$	PHI	porosity

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PROGRAM LISTING

55  
(page 57 follows)

```

C MAIN PROGRAM                                MNP 10
C *****                                 MNP 20
C IMPLICIT REAL*8(A-H,O-Z)                  MNP 30
C                                         MNP 40
C PURPOSE: TO SIMULATE VAPOUR- AND LIQUID-DOMINATED GEOTHERMAL MNP 50
C RESERVOIRS USING FINITE DIFFERENCE TECHNIQUES MNP 60
C PROGRAMMED BY CHARLES R. FAUST AND JAMES W. MERCER MNP 70
C U. S. GEOLOGICAL SURVEY                   MNP 80
C 1975-1976                                  MNP 90
C -----
C                                         MNP 100
C TO MINIMIZE CORE, DIMENSION A AND R FOR EACH PROBLEM: MNP 110
C A(NBB,MBE), R(NBB)                         MNP 120
C DIMENSION A(300,43), R(30U)                 MNP 130
C                                         MNP 140
C                                         MNP 150
C COMMON /INPUT/ PHI(20,10),XK(20,10),YK(20,10),P(20,10),H(20,10),X(MNP 160
1400),X1(40U),NP(20,10),NPP(200,2),DX(2U),DY(10),DZ(20,10),Q(20,10)MNP 170
COMMON /CONTROL/ NK,NX,NXX,NY,NYY,NB,NBD,NT,DELT,TIME,PHFWT,DF,XKC,MNP 180
ICOND,COEF,BETA,IPRT                           MNP 190
COMMON /WORK/ AX(21,10),BX(21,10),EX(21,10),AY(20,11),BY(20,11),EYMNP 200
1(20,11),TX(21,10),TXK(21,10),TY(20,11),TYK(20,11),XM(20,10),XMASS(MNP 210
220,10),EN(20,10),ENERGY(20,10),C(20,10),D(20,10),F(20,10),G(20,10)MNP 220
3,QF(20,10),XN(20,10),TN(20,10),UTP(20,10),UTH(20,10),PHIC(20,10),PMNP 230
4OLD(20,10)                                     MNP 240
COMMON /EXTRA/ MBW,MBE                         MNP 250
COMMON /CHECK/ IND(20J),INDOLD(200)            MNP 260
C -----
C                                         MNP 270
C GET DATA -                                    MNP 280
CALL GDATA(IOPT)                            MNP 290
*****                                 MNP 300
NKK=NK                                         MNP 310
C                                         MNP 320
FOR DIMENSIONING -                          MNP 330
NBBD=NBB                                     MNP 340
MBED=MBE                                     MNP 350
MNP 360
C                                         MNP 370
C CALCULATE TRANSMISSIBILITY TERMS -        MNP 380
CALL TCALC(UY,DX,DZ,XK,YK,TY,TYK,TX,TAN,XKC,NY,NX,NYY,NXX) MNP 390
*****                                 MNP 400
C                                         MNP 410
C CALCULATE BANDWIDTH -                      MNP 420
CALL MATH(NP,NY,NX,MBE,MBW)                 MNP 430
*****                                 MNP 440
C                                         MNP 450
C CHECK INITIAL PHASE CONDITIONS           MNP 460
C COMPUTE THERMODYNAMIC REGION -            MNP 470
CALL PHREG(IND)                            MNP 480
*****                                 MNP 490
DO 10 K=1,NB                                MNP 500
10 INDOLD(K)=IND(K)                         MNP 510
C                                         MNP 520

```

```

C      :CALCULATE INITIAL BALANCE -
C      CALL BALNCE(0,NKK,PCEE)
C      ****
C
C      :TIME LOOP -
C      DO 180 L=1,NT
C
C      :ITERATION LOOP -
C      DO 130 KKK=1,NK
C
C      :CALCULATE PROPERTY COEFFICIENTS -
C      CALL PRPTY(L,KKK)
C      ****
C
C      :INITIALIZE BOUNDARY ARRAYS -
C      DO 20 I=1,NX
C          AY(I,1)=0.D0
C          BY(I,1)=0.D0
C          EY(I,1)=0.D0
C          AY(I,NYY)=0.D0
C          BY(I,NYY)=0.D0
C          20 EY(I,NYY)=0.D0
C
C      DO 30 J=1,NY
C          AX(1,J)=0.D0
C          BX(1,J)=0.D0
C          EX(1,J)=0.D0
C          AX(NXX,J)=0.D0
C          BX(NXX,J)=0.D0
C          30 EX(NXX,J)=0.D0
C
C      DO 50 J=1,NY
C      DO 50 I=2,NX
C      FOR UPSTREAM WEIGHTING -
C          ALPHA=1.D0
C          IF (P(I-1,J).GT.P(I,J)) ALPHA=0.D0
C          IF (IOPT.EQ.1) GO TO 40
C      FOR MIDPOINT WEIGHTING -
C          ALPHA=DX(I)/(DX(I)+DX(I-1))
C          40 EX(I,J)=(ALPHA*XN(1,J)+(1.0-ALPHA)*XN(I-1,J))*TX(I,J)
C              AX(I,J)=(ALPHA*DTP(I,J)+(1.0-ALPHA)*DTP(I-1,J))*TXK(I,J)+(ALPHA*TMM*P(I,J)
C              +(1.0-ALPHA)*TM(I-1,J))*TX(I,J)
C          50 BX(I,J)=(ALPHA*DTH(I,J)+(1.0-ALPHA)*DTH(I-1,J))*TXK(I,J)
C
C      DO 70 I=1,NX
C      DO 70 J=2,NY
C          ALPHA=1.D0
C          IF (P(I,J-1).GT.P(I,J)) ALPHA=0.D0
C          IF (IOPT.EQ.1) GO TO 60
C          ALPHA=DY(J)/(DY(J)+DY(J-1))
C          60 EY(I,J)=(ALPHA*XN(I,J)+(1.0-ALPHA)*XN(I,J-1))*TY(I,J)

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```

AY(I,J)=(ALPHA*DTP(I,J)+(1.0-ALPHA)*DTP(I,J-1))*TYK(I,J)+(ALPHA*TMMNP1050
1(I,J)+(1.0-ALPHA)*TM(I,J-1))*TY(I,J) MNP1060
70 BY(I,J)=(ALPHA*DTH(I,J)+(1.0-ALPHA)*DTH(I,J-1))*TYK(I,J) MNP1070
C MNP1080
C FORM FINAL MATRIX EQUATION -
CALL FORMEQ(A,R,NBB,MBED) MNP1090
C ***** MNP1100
C IHALF=(MBW-1)/2 MNP1110
C MNP1120
C TRIANGULARIZE - MNP1130
CALL SOLVE(1,A,R,NBB,IHALF,NBB,MBED) MNP1140
C ***** MNP1150
C MNP1160
C SOLVE - MNP1170
CALL SOLVE(2,A,R,NBB,IHALF,NBB,MBED) MNP1180
C ***** MNP1190
C MNP1200
DO 80 K=1,NB MNP1210
X1(2*K-1)=X1(2*K-1)+R(2*K-1) MNP1220
80 X1(2*K)=X1(2*K)+R(2*K) MNP1230
IF (KKK.NE.1) GO TO 110 MNP1240
85 CONTINUE MNP1245
CALL PHREG(IND) MNP1250
C ***** MNP1260
C MNP1270
C DID A CONVERSION OCCURE? - MNP1280
DO 100 K=1,NB MNP1290

```

```

100 INDOLD(K)=IND(K) MNP1450
GO TO 105 MNP1455
110 CONTINUE MNP1460
IF (KKK.GE.4) GO TO 85 MNP1465
105 CONTINUE MNP1467
DO 120 K=1,NB MNP1470
I=NPP(K,1) MNP1480
J=NPP(K,2) MNP1490
P(I,J)=X1(2*K-1) MNP1500
120 H(I,J)=X1(2*K) MNP1510
WRITE (6,190) KKK MNP1520

```

```

C      CALCULATE MASS AND HEAT BALANCE -          MNF1530
C      CALL BALANCE(1,KKK,PCEE)                   MNF1540
C      *****                                     MNF1550
C      ERR=DABS(PCEE)                           MNF1560
C      CHECK ERROR OF ENERGY BALANCE TO DETERMINE NUMBER OF ITERATIONS - MNF1580
C      IF (ERR.LT.0.02) GO TO 140                MNF1590
130  CONTINUE                                    MNF1600
      GO TO 160                                    MNF1610
140  CONTINUE                                    MNF1620
      DO 150 I=1,NX                            MNF1630
      DO 150 J=1,NY                            MNF1640
      XMASS(I,J)=XM(I,J)                      MNF1650
150  ENERGY(I,J)=EN(I,J)                      MNF1660
160  CONTINUE                                    MNF1670
C
C      END ITERATION LOOP                      MNF1680
      DO 170 K=1,NBB                         MNF1690
170  X(K)=X1(K)                                MNF1700
      TIME=TIME+DELT                          MNF1710
C
C      PRINT RESULTS -                        MNF1730
      CALL PDATA(P,H,TIME,L,NY,NX)            MNF1740
C      *****                                     MNF1750
180  CONTINUE                                    MNF1760
C
C      END TIME LOOP                          MNF1770
      STOP                                     MNF1780
C
190  FORMAT (//15X,16HITERATION NUMBER,I5)    MNF1790
      END                                     MNF1800
                                         MNF1810
                                         MNF1820
                                         MNF1830-

```

```

SUBROUTINE GDATA(IOPT) DAT 10
C #####*#####*#####*#####*#####* DAT 20
C IMPLICIT REAL*8(A-H,O-Z) DAT 30
C
C CALLED FROM MAIN DAT 40
C PURPOSE: TO READ AND WRITE PROBLEM INFORMATION DAT 50
C -----
C DIMENSION TITLE(20) DAT 70
C
C COMMON /INPUT/ PHI(20,10),XK(20,10),YK(20,10),P(20,10),H(20,10),X(DAT 100
1400),X1(400),NP(20,10),NPP(200,2),DX(20),DY(10),DZ(20,10),Q(20,10)DAT 110
C COMMON /CONTROL/ NK,NX,NXX,NY,NYY,NB,NBE,NT,DELT,TIME,PHFWT,DF,XKC,DAT 120
1COND,COEF,BETA,IPRT DAT 130
C COMMON /WORK/ AX(21,10),BX(21,10),EX(21,10),AY(20,11),BY(20,11),EYDAT 140
1(20,11),TX(21,10),TXK(21,10),TY(20,11),TYK(20,11),XM(20,10),XMASS(DAT 150
220,10),EN(20,10),ENERGY(20,10),C(20,10),D(20,10),F(20,10),G(20,10)DAT 160
3,QH(20,10),XN(20,10),TM(20,10),DTP(20,10),DTH(20,10),PHJC(20,10),PDAT 170
4OLD(20,10) DAT 180
C COMMON /EXTRA/ MBW,MBE DAT 190
C -----
C READ AND WRITE UNITS - DAT 200
C NR=5 DAT 210
C NW=6 DAT 220
C
C INITIALIZE SIMULATION TIME - DAT 230
C TIME=0.00 DAT 240
C
C READ AND WRITE TITLE OF PROBLEM - DAT 250
C READ (NR,280) TITLE DAT 260
C WRITE (NW,290) DAT 270
C WRITE (NW,300) TITLE DAT 280
C
C READ AND WRITE FINITE DIFFERENCE INFORMATION - DAT 290
C READ (NR,310) NX,NY,NB,NK,NT,NS,MHE,IUPT,IPRT DAT 300
C NX - NUMBER OF COLUMNS (X-DIRECTION) DAT 310
C NY - NUMBER OF ROWS (Y-DIRECTION) DAT 320
C NB - NUMBER OF NON-ZERO BLOCKS DAT 330
C NK - MAXIMUM NUMBER OF NEWTON ITERATIONS DAT 340
C NT - MAXIMUM NUMBER OF TIME STEPS DAT 350
C NS - NUMBER OF SOURCES DAT 360
C MHE - ESTIMATED BANDWIDTH DAT 370
C IOPT - READ 1 FOR UPSHIFT WEIGHTING ON PRESSURE;
C           2 FOR MIDPOINT WEIGHTING DAT 380
C IPRT - NUMBER OF TIME STEPS BETWEEN PRINTING THERMO DATA DAT 390
C WRITE (NW,320) DAT 400
C WRITE (NW,330) NX,NY,NB,NK,NT,NS,MBE,IUPT,IPRT DAT 410
C
C NXX=NX+1 DAT 420
C NYY=NY+1 DAT 430
C NBE=2*NB DAT 440
C NBE - TOTAL NUMBER OF EQUATIONS DAT 450

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C          : DAT 530
C TIME PARAMETERS - DAT 540
C READ (NR,340) DELT DAT 550
C DELT - TIME STEP IN SECONDS DAT 560
C WRITE (NW,370) DELT DAT 570
C DAT 580
C DATA CODES - DAT 590
C READ (NR,310) KOD1,KOD2,KOD3,KOD4,KOD5,KOD6,KOD7,KOD8,KOD9 DAT 600
C KOD1 - READ 1 IF X-SPACING IS CONSTANT DAT 610
C KOD2 - READ 1 IF Y-SPACING IS CONSTANT DAT 620
C KOD3 - READ 1 IF INITIAL PRESSURE IS CONSTANT DAT 630
C KOD4 - READ 1 IF INITIAL ENTHALPY IS CONSTANT DAT 640
C KOD5 - READ 1 IF X-PERMEABILITY IS CONSTANT DAT 650
C KOD6 - READ 1 IF Y-PERMEABILITY IS CONSTANT DAT 660
C KOD7 - READ 1 IF INITIAL POROSITY IS CONSTANT DAT 670
C KOD8 - READ 1 IF RESERVOIR THICKNESS IS CONSTANT DAT 680
C KOD9 - READ 1 IF TEMPERATURE IS READ IN PLACE OF ENTHALPY DAT 690
C WRITE (NW,260) KOD1,KOD2,KOD3,KOD4,KOD5,KOD6,KOD7,KOD8,KOD9 DAT 700
C DAT 710
C SPACING - DAT 720
C IF (KOD1.EQ.1) GO TO 10 DAT 730
C READ (NH,340) (DX(I),I=1,NX) DAT 740
C GO TO 30 DAT 750
10 READ (NR,340) DX1 DAT 760
DO 20 I=1,NX DAT 770
20 DX(I)=DX1 DAT 780
30 WRITE (NW,350) DAT 790
    WRITE (NW,390) (DX(I),I=1,NX) DAT 800
C DAT 810
C IF (KOD2.EQ.1) GO TO 40 DAT 820
C READ (NH,340) (DY(J),J=1,NY) DAT 830
C GO TO 60 DAT 840
40 READ (NR,340) DY1 DAT 850
DO 50 J=1,NY DAT 860
50 DY(J)=DY1 DAT 870
60 WRITE (NW,360) DAT 880
    WRITE (NW,390) (DY(J),J=1,NY) DAT 890
C DAT 900
C PRESSURE - DAT 910
C WRITE (NW,380) DAT 920
C CALL READ(P,KOD3,NY,NX) DAT 930
C **** DAT 940
C DAT 950
C ENTHALPY - DAT 960
C IF (KOD9.EQ.1) WRITE (NW,270) DAT 970
C IF (KOD9.NE.1) WRITE (NW,400) DAT 980
C CALL READ(H,KOD4,NY,NX) DAT 990
C **** DAT 1000
C IF (KOD9.NE.1) GO TO 100 DAT 1010
C DAT 1020
C TEMPERATURES READ (COMPRESSED WATER REGION ASSUMED) DAT 1030
C COMPUTE ENTHALPY BY NEWTON-RAPHSON METHOD DAT 1040

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130 WRITE (NW,470) I,J,Q(I,J) DAT1570
C
C      ROCK PROPERTIES -
140 READ (NR,340) XKC,CONU,PHFWT,DF,BETA DAT1580
C      XKC - MEDIUM THERMAL CONDUCTIVITY DAT1590
C      COND - CONFINING BED THERMAL CONDUCTIVITY DAT1600
C      PHFWT - ROCK ENTHALPY DERIVATIVE W.R.T. TEMPERATURE DAT1610
C      DF - ROCK DENSITY DAT1620
C      BETA - ROCK COMPRESSIBILITY DAT1630
C      WRITE (NW,480) XKC,CONU,PHFWT,DF,BETA DAT1640
C
C      COMPUTE COEFFICIENT FOR CONDUCTIVE LEAKAGE -
IF (COND.LT.0.1) GO TO 150 DAT1650
COEF=DF*PHFWT/COND DAT1660
GO TO 160 DAT1670
150 COEF=0.D0
160 CONTINUE DAT1680
C
C      NUMBER GRID BLOCKS -
C      NP(I,J) - SEQUENCE NUMBERING OF BLOCKS I,J DAT1690
IBN=0 DAT1700
DO 190 I=1,NX DAT1710
DO 180 J=1,NY DAT1720
IF (DZ(I,J).LT.0.001) GO TO 170 DAT1730
IBN=IBN+1 DAT1740
NP(I,J)=IBN DAT1750
GO TO 180 DAT1760
170 NP(I,J)=0 DAT1770
180 CONTINUE DAT1780
190 CONTINUE DAT1790
WRITE (NW,240) DAT1800
DO 200 J=1,NY DAT1810
200 WRITE (NW,250) (NP(I,J),I=1,NX) DAT1820
C
C      COMPUTE I,J FOR EACH SEQUENTIAL BLOCK NUMBER -
DO 220 I=1,NX DAT1830
DO 220 J=1,NY DAT1840
IJ=NP(I,J) DAT1850
IF (IJ.EQ.0) GO TO 220 DAT1860
NPP(IJ,1)=I DAT1870
NPP(IJ,2)=J DAT1880
GO TO 220 DAT1890
220 CONTINUE DAT1900
C
C      INITIALIZE SOLUTION VECTOR -
DO 230 K=1,NB DAT1910
II=NPP(K,1) DAT1920
JJ=NPP(K,2) DAT1930
X(2*K-1)=P(II,JJ) DAT1940
X1(2*K-1)=P(II,JJ) DAT1950
X(2*K)=H(II,JJ) DAT1960
X1(2*K)=H(II,JJ) DAT1970
X(2*K)=H(II,JJ) DAT1980
X1(2*K)=H(II,JJ) DAT1990

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230 CONTINUE DAT2100
      RETURN DAT2110
C DAT2120
240 FORMAT (11X,12HGRID NUMBERS/11X,12(1H-)) DAT2130
250 FORMAT (11X,16I5) DAT2140
260 FORMAT (/////11X,5HCODES/11X,5(1H-)/11X,9I5) DAT2150
270 FORMAT (/////,11X,19HINITIAL TEMPERATURE/11X,19(1H-)/) DAT2160
280 FORMAT (20A4) DAT2170
290 FORMAT (1H1,///35X,25HSTEAM-WATER FLOW ANALYSIS//) DAT2180
300 FORMAT (11X,70(1H*)//11X,20A4//11X,70(1H*)///) DAT2190
310 FORMAT (16I5) DAT2200
320 FORMAT (/11X,22HFINITE DIFFERENCE DATA/11X,22(1H-)/) DAT2210
330 FORMAT (1H ,10X,11HNUMBER OF -,2X,7HCOLUMNS,I23/21X,1H-,2X,4HRGWS,DAT2220
     1I26/21X,1H-,2X,15HNON-ZERO BLOCKS,I15/21X,1H-,2X,18HMAXIMUM ITERATUAT2230
     2IONS,I12/21X,1H-,2X,18HMAXIMUM TIME STEPS,I12/21X,1H-,2X,7HSOURCESDAT2240
     3,I23/21X,1H-,2X,19HESTIMATED BANDWIDTH,I11/21X,1H-,2X,16HWEIGHTINGDAT2250
     4 OPTION,I14/21X,1H-,2X,15HPRINTING OPTION,I15///) DAT2260
340 FORMAT (8G10.0) DAT2270
350 FORMAT (/////,11X,22HSPACING IN X-DIRECTION/11X,22(1H-)/) DAT2280
360 FORMAT (/////,11X,22HSPACING IN Y-DIRECTION/11X,22(1H-)/) DAT2290
370 FORMAT (////,11X,15HTIME PARAMETERS/11X,15(1H-)//11X,28HINITIAL TIMDAT2300
     1E STEP IN SECONDS,G22.8/) DAT2310
380 FORMAT (/////,11X,16HINITIAL PRESSURE/11X,16(1H-)/) DAT2320
390 FORMAT ((11X,8(G12.5,2X))) DAT2330
400 FORMAT (/////,11X,16HINITIAL ENTHALPY/11X,16(1H-)/) DAT2340
410 FORMAT (/////,11X,14HX-PERMEABILITY/11X,14(1H-)/) DAT2350
420 FORMAT (/////,11X,14HY-PERMEABILITY/11X,14(1H-)/) DAT2360
430 FORMAT (/////,11X,16HINITIAL POROSITY/11X,16(1H-)/) DAT2370
440 FORMAT (/////,11X,19HRESERVOIR THICKNESS/11X,19(1H-)/) DAT2380
450 FORMAT (/////,11X,7HSOURCES/11X,7(1H-)/) DAT2390
460 FORMAT (2I5,G15.0) DAT2400
470 FORMAT (11X,2I5,G15.8) DAT2410
480 FORMAT (/11X,15HROCK PROPERTIES/11X,15(1H-)/11X,28HMEDIUM THERMAL DAT2420
     1CONDUCTIVITY-G17.5/11X,35HCONFINING BED THERMAL CONDUCTIVITY-G10.30AT2430
     2/11X,25HROCK ENTHALPY DERIVATIVE-G20.0/11X,13HROCK DENSITY-G32.5/10A12440
     31X,1ROCK COMPRESSIBILITY-,G24.5/) DAT2450
      END DAT2460-

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```

C      SUBROUTINE READ(DUM,KODE,NY,NX)          REC   10
C      *****                                         REC   20
C      IMPLICIT REAL*8(A-H,O-Z)                   REC   30
C
C      CALLED FROM GDATA                         REC   40
C      PURPOSE: TO READ TWO-DIMENSIONAL ARRAYS    REC   50
C      -----
C      DIMENSION DUM(20,10)                      REC   70
C      -----
C      NR=5                                       REC   80
C      NW=6                                       REC   90
C      IF (KODE.EQ.1) GO TO 20
C      DO 10 J=1,NY
C      10 READ (NR,70) (DUM(I,J),I=1,NX)        REC 100
C          GO TO 40
C      20 READ (NR,70) DUM1
C          DO 30 J=1,NY
C          DO 30 I=1,NX
C          30 DUM(I,J)=DUM1
C          40 DO 50 J=1,NY
C          50 WRITE (NW,60) (DUM(I,J),I=1,NX)
C              RETURN
C
C      60 FORMAT (/((11X,8(G12.5,ZX)))           REC 110
C      70 FORMAT (8G10.0)                          REC 120
C      END                                         REC 130
C
C      REC 140
C      REC 150
C      REC 160
C      REC 170
C      REC 180
C      REC 190
C      REC 200
C      REC 210
C      REC 220
C      REC 230
C      REC 240
C      REC 250
C      REC 260-

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SUBROUTINE TCALC(DY,DX,DZ,XK,YK,TY,TK,XKC,NY,NX,NYY,NXX) TCA 10
C ##### IMPLICIT REAL*8(A-H,O-Z) TCA 20
C
C CALLED FROM MAIN TCA 30
C PURPOSE: TO COMPUTE TRANSMISSIBILITY TERMS - TCA 40
C -----
C DIMENSION DY(10), DX(20), DZ(20,10), XK(20,10), YK(20,10), TY(20,1) TCA 70
C 11, TK(10,11), TX(21,10), TXK(21,10) TCA 80
C -----
C COMPUTE TRANSMISSIBILITY TERMS IN THE X-DIRECTION - TCA 100
C DO 10 J=1,NY TCA 110
C IF ONLY ONE COLUMN, SKIP CALCULATIONS - TCA 120
C IF (NX.EQ.1) GO TO 20 TCA 130
C DO 10 I=2,NX TCA 140
C TXC=DY(J)*DZ(I-1,J)/DX(I-1) TCA 150
C TXD=DY(J)*DZ(I,J)/DX(I) TCA 160
C TXA=TXC*XK(I-1,J) TCA 170
C TXB=TXD*XK(I,J) TCA 180
C TTI=TXA+TXB TCA 190
C IF (TTT.EQ.0.0) GO TO 10 TCA 200
C PERMEABILITY TERM - TCA 210
C TX(I,J)=2.0*TXA*TXB/TTI TCA 220
C HEAT CONDUCTION TERM - TCA 230
C TXK(I,J)=2.0*TXC*TXD/(TXC+TXD)*XKC TCA 240
C 10 CONTINUE TCA 250
C 20 CONTINUE TCA 260
C
C COMPUTE TRANSMISSIBILITY TERMS IN THE Y-DIRECTION - TCA 270
C DO 30 I=1,NX TCA 280
C IF (NY.EQ.1) GO TO 40 TCA 290
C DO 30 J=2,NY TCA 300
C TYC=DX(I)*DZ(I,J-1)/DY(J-1) TCA 310
C TYD=DX(I)*DZ(I,J)/DY(J) TCA 320
C TYA=TYC*YK(I,J-1) TCA 330
C TYB=TYD*YK(I,J) TCA 340
C TTT=TYA+TYB TCA 350
C IF (TTT.EQ.0.0) GO TO 30 TCA 360
C TY(I,J)=2.0*TYA*TYB/TTI TCA 370
C TYK(I,J)=2.0*TYC*TYD/(TYC+TYD)*XKC TCA 380
C 30 CONTINUE TCA 390
C 40 CONTINUE TCA 400
C
C SET TRANSMISSIBILITY OF BOUNDARY BLOCKS (REGULAR RECTANGULAR MESH) TCA 410
C TO ZERO (NO-FLOW) - TCA 420
C DO 50 I=1,NX TCA 430
C TY(I,1)=0.00 TCA 440
C TYK(I,1)=0.00 TCA 450
C TY(I,NYY)=0.00 TCA 460
C 50 TYK(I,NYY)=0.00 TCA 470
C

```

```

DO 60 J=1,NY          TCA 530
TX(1,J)=0.D0          TCA 540
TXK(1,J)=0.D0          TCA 550
TX(NXX,J)=0.D0          TCA 560
60 TXK(NXX,J)=0.D0          TCA 570
C
C      SET TRANSMISSIBILITY OF BOUNDARY BLOCKS (EXTERNAL AND INTERNAL,
C      IRREGULAR MESH) TO ZERO -
DO 80 I=1,NX          TCA 580
DO 70 J=1,NY          TCA 590
IF (DZ(I,J).GT.0.D0) GO TO 70          TCA 600
TX(I+1,J)=0.D0          TCA 610
TXK(I+1,J)=0.D0          TCA 620
TY(I,J+1)=0.D0          TCA 630
TYK(I,J+1)=0.D0          TCA 640
TXK(I,J)=0.D0          TCA 650
TX(I,J)=0.D0          TCA 660
TYK(I,J)=0.D0          TCA 670
TY(I,J)=0.D0          TCA 680
70 CONTINUE          TCA 690
80 CONTINUE          TCA 700
RETURN          TCA 710
END          TCA 720
TCA 730
TCA 740
TCA 750-

```

```

C      SUBROUTINE MATH(NP,NY,NX,MBE,MBW)
C      *****
C      IMPLICIT REAL*8(A-H,O-Z)
C
C      CALLED FROM MAIN
C      PURPOSE: COMPUTE MATRIX BANDWIDTH
C      -----
C      DIMENSION N(5), NP(20,10)
C      -----
C
C      COMPUTE MAXIMUM DIFFERENCE BETWEEN SEQUENTIAL BLOCK NUMBERS
C      OF ADJACENT BLOCKS -
C      MM=0
C      DO 80 I=1,NA
C      DO 70 J=1,NY
C      DO 10 KK=1,4
C      10 N(KK)=0
C      I1=I-1
C      I2=I+1
C      J1=J-1
C      J2=J+1
C      IF (I1.EQ.0) GO TO 20
C      N(1)=NP(I1,J)
C      20 CONTINUE
C      IF (I2.GT.NX) GO TO 30
C      N(3)=NP(I2,J)
C      30 CONTINUE
C      IF (J1.EQ.0) GO TO 40
C      N(2)=NP(I,J1)
C      40 CONTINUE
C      IF (J2.GT.NY) GO TO 50
C      N(4)=NP(I,J2)
C      50 N(5)=NP(I,J)
C      IF (N(5).EQ.0) GO TO 70
C      DO 60 K=1,4
C      IF (N(K).EQ.0) GO TO 60
C      C      COMPUTE DIFFERENCE -
C      NN=N(5)-N(K)
C      IF (NN.LT.0) NN=-NN
C      C      COMPUTE MAXIMUM DIFFERENCE -
C      IF (NN.GT.MM) MM=NN
C      60 CONTINUE
C      70 CONTINUE
C      80 CONTINUE
C      C      COMPUTE BANDWIDTH -
C      MBW=2*(2*MM+1)+1
C      PRINT 90, MBW,MBE
C      RETURN
C
C      90 FORMAT (//11X,'MAXIMUM BANDWIDTH IS ',I4,2X,'COMPARED TO ESTIMATED
C      1 BANDWIDTH OF ',I4//)
C      END

```

```

C SUBROUTINE PRPTY(L,KKK)
C *****IMPLICIT REAL*8(A-H,O-Z)
C CALLED FROM MAIN
C PURPOSE: TO COMPUTE THERMODYNAMIC PROPERTIES
C -----
C DIMENSION SW1(20,10), TEMP2(20,10), DEN2(20,10)
C DIMENSION CQ(20,10)
C
C COMMON /INPUT/ PHI(20,10),XK(20,10),YK(20,10),P(20,10),H(20,10),X(PRP 110
1400),X1(400),NP(20,10),NPP(200,2),DX(20),DY(10),DZ(20,10),G(20,10)PRP 120
C COMMON /CONTROL/ NK,NX,NXX,NY,NYY,NB,NBB,NT,DELT,TIME,PHFWT,UF,XKC,PRP 130
1COND,COEF,BETA,IPRT
C COMMON /WORK/ AX(21,10),BX(21,10),EX(21,10),AY(20,11),BY(20,11),EX(PRP 150-
1(20,11),TX(21,10),TXK(21,10),TY(20,11),TYK(20,11),XM(20,10),XMASS(PRP 160
220,10),EN(20,10),ENERGY(20,10),C(20,10),D(20,10),F(20,10),G(20,10)PRP 170
3,QH(20,10),XN(20,10),TM(20,10),DTP(20,10),DTH(20,10),PHIC(20,10),PPRP 180
4OLD(20,10)
C COMMON /CHECK/ IND(200),INDOLD(200)
C -----
C STATEMENT FUNCTIONS FOR THERMODYNAMIC PROPERTIES
C
C F1(PX,HX)=1.00207+4.42607U-4*PX-5.47456D-5*HX+5.02875D-7*HX*PX-1.2PRP 250
14791D-7*HX*HX
C F2(PX,HX)=-2.41231+2.56222D-1*HX-9.31415D-3*PX*PX-2.256HD-5*HX*HX PRP 270
C F3(PX,HX)=-2.26162D-5+v.0438441D0*PX-1.79088D-5*PX*HX+3.69276D-3*PPRF 280
1X*PX*PX*PX+5.17644D-13*HX*HX*HX*PX
C F4(PX,HX)=-374.669D0+4/.9921D0*PX-0.633606D0*PX*PX+7.39386D-5*PX*HX*HX PRP 300
1X-3.3372D6/HX/HX/PX/PX+0.0357154D0/PX/PX/PX-1.1725D-9*HX*HX*HX*PX-PRP 310
22.26861D15/HX/HX/HX/HX
C
C F1 - DENSITY, COMPRESSED WATER REGION
C F2 - TEMPERATURE, COMPRESSED WATER REGION
C F3 - DENSITY, SUPER HEATED STEAM REGION
C F4 - TEMPERATURE, SUPER HEATED STEAM REGION
C NKK=NK
C IF (L.NE.1) GO TO 20
C IF (KKK.NE.1) GO TO 20
C
C INITIALIZE ARRAYS -
DO 10 I=1,NX
DO 10 J=1,NY
XN(I,J)=0.D0
TM(I,J)=0.D0
C(I,J)=0.D0
D(I,J)=0.D0
F(I,J)=0.D0
G(I,J)=0.D0
QH(I,J)=0.D0
DTP(I,J)=0.D0

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DTF(I,J)=0.00          PRP 530
SW1(I,J)=0.00          PRP 540
DEN2(I,J)=0.00          PRP 550
TEMP2(I,J)=0.00          PRP 560
10 CONTINUE             PRP 570
20 CONTINUE             PRP 580
DO 80 I=1,NX            PRP 590
  DO 80 J=1,NY            PRP 600
    IF (DZ(I,J).EQ.0.00) GO TO 80
    PP=P(I,J)
    HH=H(I,J)
    PP=PP*0.1**7
    HH=HH*0.1**7
C
    K=NP(I,J)
    IF (IND(K)) 30,40,50
30 CONTINUE             PRP 610
C
C   COMPRESSED WATER REGION -
C
      DW=F1(PP,HH)           PRP 620
      TEMP=F2(PP,HH)          PRP 630
      PDWP=4.42607D-11+5.02875D-14*HH          PRP 640
      PDWH=-5.47456D-12+5.02875D-14*PP-2.49582D-14*HH          PRP 650
      PTWP=-1.86283D-9*PP          PRP 660
      PTWH=2.56222D-8-4.5136D-12*HH          PRP 670
C   WATER VISCOSITY -
C   1967 ASME STEAM TABLES FORMULA (P. 74) -
      VW=1.00-6*(241.4*10.***(247.8/(TEMP+133.15)))          PRP 680
C
C   COMPUTE EQUATION COEFFICIENTS -
C   SPACE -
      XN(I,J)=DW/VW          PRP 690
      TM(I,J)=DW*HH/VW*10.**/          PRP 700
      UTF(I,J)=PTWH          PRP 710
      DTF(I,J)=PTWP          PRP 720
C   TIME -
      C(I,J)=PDWP*PHI(I,J)*HH*10.**7+PHFWT*PIWP*DF*(1.0-PHI(I,J))          PRP 730
      C(I,J)=C(I,J)+DW*HH*1.07*BETA*PHIO(I,J)-PHIO(I,J)*BETA*DF*PHFWT*T*PRP 740
1IMP
      D(I,J)=PDWH*PHI(I,J)*HH*10.**7+PHFWT*PIWH*DF*(1.0-PHI(I,J))+DW*PHI*PRP 750
1(I,J)
      F(I,J)=PHI(I,J)*PDWP+U**BETA*PHIO(I,J)          PRP 760
      G(I,J)=PHI(I,J)*PDWH          PRP 770
C
      SW1(I,J)=1.00          PRP 780
      TEMP2(I,J)=TEMP          PRP 790
      DEN2(I,J)=DW          PRP 800
C   HEAT DISCHARGE -
      QH(I,J)=Q(I,J)*HH*10.**7          PRP 810
      GO TO 60          PRP 820

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40 CONTINUE PRP1070
C PRP1080
C PRP1090
C TWO-PHASE REGION - PRP1100
  HW=730.984+129.239*PP-10.0333*PP*PP+0.39881*PP*PP*PP
  1 -99.0697/PP+12.9267/PP/PP-0.628359/PP/PP/PP PRP1110
  HS=2822.82-39.952/PP+2.54342/PP/PP-0.938879*PP*PP PRP1120
  TEMP=F2(PP,HW) PRP1130
  DW=F1(PP,HW) PRP1140
  DS=F3(PP,HS) PRP1150
C WATER SATURATION - PRP1160
  SW=DS*(HS-HW)/(HH*(DW-DS)-(HW*DWS*DS)) PRP1170
  SST=1.0-SW PRP1180
  SW0=SW PRP1190
C VISCOSITIES - PRP1200
C 1967 ASME STEAM TABLES FORMULA (P. 74) - PRP1210
  VW=1.0E-06*(241.4*10.**247.8/(TEMP+133.15)) PRP1220
  VS=1.0E-06*(407*TEMP+80.4) PRP1230
C RELATIVE PERMEABILITY (WATER) - PRP1240
  XKW=((SW-0.35)/0.65)**4 PRP1250
C RELATIVE PERMEABILITY (STEAM) - PRP1260
  XKS=(1.0-((SW-0.35)/0.65)**2)*(1.0-((SW-0.35)/0.65))**2 PRP1270
  IF (SW.LT.0.3) XKW=0.0 PRP1280
  IF (SW.LT.0.3) XKS=1.0 PRP1290
  IF (SW.GT.0.95) XKW=1.0 PRP1300
  IF (SW.GT.0.95) XKS=0.0 PRP1310
  DEN=DW*SW+SST*DS PRP1320
  HHH=HH*1.001D0 PRP1330
  SW=DS*(HS-HHH)/(HHH*(DW-DS)-(HW*DWS*DS)) PRP1340
  SST=1.0-SW PRP1350
  DEN1=DW*SW+DS*SST PRP1360
  PDWH=(DEN1-DEN)/(HHH-HH)*1.D-7 PRP1370
  PPP=PP*.999D0
  HW=730.984+129.239*PPP-10.0333*PPP*PPP+0.39881*PPP*PPP*PPP
  1 -99.0697/PPP+12.9267/PPP/PPP-0.628359/PPP/PPP/PPP PRP1400
  HS=2822.82-39.952/PPP+2.54342/PPP/PPP-0.938879*PPP*PPP PRP1410
  T1=F2(PPP,HHW) PRP1420
  DDW=F1(PPP,HHW) PRP1430
  DDS=F3(PPP,HHS)
  SW=DDS*(HHS-HH)/(HH*(DDW-DDS)-(HHW*DDW-HHS*DDS)) PRP1440
  SST=1.0-SW PRP1450
  DEN1=DDW*SW+DDS*SST PRP1460
  PDWP=(DEN1-DEN)/(PPP-PP)*1.D-7 PRP1470
  TEMP1=T1 PRP1480
  PTWP=(TEMP1-TEMP)/(PPP-PP)*1.D-7 PRP1490
  SW=SW0 PRP1500
C SPACE - PRP1510
  XN(I,J)=XKW*DWS/VW+XKS*DWS/VS PRP1520
  TM(I,J)=(XKW*DWS/HW/VW+XKS*DWS*HS/VS)*10.**7 PRP1530
  DTP(I,J)=PTWP PRP1540
  DTW(I,J)=0.0 PRP1550
  PRP1560

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C TIME -
C(I,J)=PDWP*PHI(I,J)*HH*10.**7+PHFWT*PTWP*DF*(1.0-PHI(I,J)) PRP1570
C(I,J)=C(I,J)+DEN*HH*1.D7*BETA*PHIO(I,J)-PHIO(I,J)*BETA*DF*PHFWT*TPRP1590
1EMP PRP1600
D(I,J)=PDWH*PHI(I,J)*HH*10.**7+DEN*PHI(I,J) PRP1610
F(I,J)=PHI(I,J)*PDWP+DEN*BETA*PHIO(I,J) PRP1620
G(I,J)=PHI(I,J)*PDWH PRP1630
PRP1640
C SW1(I,J)=SW PRP1650
TEMP2(I,J)=TEMP PRP1660
DEN2(I,J)=DEN PRP1670
QH(I,J)=Q(I,J)*(HW+(HS-HW)*XKS/(XKS+VS*XKW/VW*DWS))*10.**7 PRP1680
GO TO 60 PRP1690
50 CONTINUE PRP1700
PRP1710
C
C SUPER-HEATED STEAM REGION -
DS=F3(PP,HH) PRP1730
PDWH=-1.79088D-12*PP+1.552932D-19*HH*HH*PP PRP1740
PDWP=4.38441D-9-1.79088D-12*HH+1.477104D-14*PP*PP*PP+5.17644D-20*HPRP1810
1H*HH*HH PRP1820
HW=730.984+129.239*PP-10.0333*PP*PP+0.39881*PP*PP*PP
1 -99.0697/PP+12.9267/PP/PP-0.628359/PP/PP/PP
HS=2822.82-39.952/PP+2.54342/PP/PP-0.938879*PP*PP
TT1=F4(PP,HH)
TT2=F4(PP,HS)
TT3=F2(PP,HW)
TEMP=TT1-TT2+TT3
HHH=HH*1.001D0
TT1=F4(PP,HHH)
TEMPH=TT1-TT2+TT3
PPP=PP*0.999D0
HW =730.984+129.239*PPP-10.0333*PPP*PPP+0.39881*PPP*PPP*PPP
1 -99.0697/PPP+12.9267/PPP/PPP-0.628359/PPP/PPP/PPP
HS=2822.82-39.952/PPP+2.54342/PPP/PPP-0.938879*PPP*PPP
TT1=F4(PPP,HH)
TT2=F4(PPP,HS)
TT3=F2(PPP,HW)
TEMP=TT1-TT2+TT3
PTWH=(TEMPH-TEMP)/(HHH-HH)*1.D-7
PTWP=(TEMP-TEMP)/(PPP-PP)*1.D-7
C STEAM VISCOSITY - PRP1830
C 1967 ASME STEAM TABLES FORMULA (P. 74) - PRP1840
VS=1.0E-06*(.407*TEMP+80.4) PRP1850
PRP1860
C SPACE - PRP1870
XN(I,J)=DS/VS PRP1880
TM(I,J)=DS*HH/VS*10.**7 PRP1890
DTH(I,J)=PTWH PRP1900
DTP(I,J)=PTWP PRP1910
C TIME - PRP1920
C(I,J)=PDWP*PHI(I,J)*HH*10.**7+PHFWT*PTWP*DF*(1.0-PHI(I,J)) PRP1930

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C(I,J)=C(I,J)+DS*HH*1.07*BETA*PHIO(I,J)-PHIO(I,J)*BETA*DF*PHFWT*TEPRP1940
1IMP :                                              PRP1950
D(I,J)=PDWH*PHI(I,J)*HH*10.**7+PHFWT*PHWH*DF*(1.0-PHI(I,J))+DS*PHIPRP1960
1(I,J)                                              PRP1970
F(I,J)=PHI(I,J)*PDWP+DS*BETA*PHIO(I,J)          PRP1980
G(I,J)=PHI(I,J)*PDWH                           PRP1990
C                                              PRP2000
SW1(I,J)=0.00                                     PRP2010
TEMP2(I,J)=TEMP                                     PRP2020
DEN2(I,J)=DS                                      PRP2030
QH(I,J)=Q(I,J)*HH*10.**7                         PRP2040
60 CONTINUE                                         PRP2050
IF (COEF.LT.0.1) GO TO 80                         PRP2060
C                                              PRP2070
C TRANSIENT CONDUCTIVE HEAT LEAKAGE ONLY -        PRP2080
IF (KKK.NE.1) GO TO 70                         PRP2090
IC=L+KKK                                         PRP2100
II=NP(I,J)                                       PRP2110
CALL VERTCD(DELT,TEMP,COEF,COND,CQQ,II,IC)      PRP2120
*****                                                 PRP2130
CQQ=CQQ*DX(I)*DY(J)                            PRP2140
CQ(I,J)=CQQ                                     PRP2150
70 CONTINUE                                         PRP2160
QH(I,J)=QH(I,J)+CQ(I,J)                         PRP2170
80 CONTINUE                                         PRP2180
IF (KKK.NE.1) GO TO 120                         PRP2190
C PRINT THERMO DATA EVERY IPRT TIME STEP -       PRP2200
IF (MOD(L,IPRT).NE.0) GO TO 120                 PRP2210
PRINT 130                                         PRP2220
DO 90 J=1,NY                                     PRP2230
PRINT 140, (SW1(I,J),I=1,NX)                   PRP2240
90 CONTINUE                                         PRP2250
PRINT 150                                         PRP2260
DO 100 J=1,NY                                    PRP2270
PRINT 140, (TEMP2(I,J),I=1,NX)                  PRP2280
100 CONTINUE                                         PRP2290
PRINT 160                                         PRP2300
DO 110 J=1,NY                                    PRP2310
PRINT 140, (DEN2(I,J),I=1,NX)                  PRP2320
110 CONTINUE                                         PRP2330
120 CONTINUE                                         PRP2340
RETURN                                           PRP2350
C                                              PRP2360
130 FORMAT (//11X,'WATER SATURATIONS'/11X,17(1H-)//) PRP2370
140 FORMAT ((11X,8(G12.5,2X)))                  PRP2380
150 FORMAT (//11X,'TEMPERATURES'/11X,12(1H-)//)    PRP2390
160 FORMAT (//11X,'DENSITY'/11X,7(1H-)//)        PRP2400
END                                              PRP2410-

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C      SUBROUTINE VERTCD(DEL1,TT,COEF,COND,CW,II,IC)          VRT  10
C      *****
C      IMPLICIT REAL*8(A-H,O-Z)          VRT  20
C
C      CALLED FROM PRPTY          VRT  30
C      PURPOSE: TO COMPUTE VERTICAL CONDUCTIVE LEAKAGE THROUGH THE          VRT  60
C                  CONFINING BED BY THE FINITE ELEMENT METHOD          VRT  70
C
C      -----
C      DIMENSION A(9), B(9), C(9), D(9), F(9), DTEMP(200,11), DELZ(200*10)          VRT  80
C      1), TI(200), DDTEMP(200,11)          VRT 100
C      -----
C      -----
C      COMPUTE DELTA Z ( ON FIRST TIME STEP) -          VRT 110
C      CTHK1=102300.00          VRT 120
C      IF (IC.NE.2) GO TO 30          VRT 130
C      NNZ=11          VRT 140
C      NNZ - NUMBER OF NODES IN Z-DIRECTION          VRT 150
C      COMPUTE DELZ (BASED ON TEN ELEMENTS AND DOUBLING THE SIZE OF EACH          VRT 160
C      ELEMENT) -          VRT 170
C      NNL=NNZ-1          VRT 180
C      NNL - NUMBER OF ELEMENTS IN Z-DIRECTION          VRT 190
C      DELZ(11,NNL)=CTHK1/1023.00          VRT 200
C      SUM=DELZ(11,NNL)          VRT 210
C      JSTOP=NNL-2          VRT 220
C      DO 10 JJ=1,JSTOP          VRT 230
C      DELZ(II,NNL-JJ)=2.*DELZ(II,NNL-JJ+1)          VRT 240
C      10 SUM=SUM+DELZ(II,NNL-JJ)          VRT 250
C      DELZ(II,1)=CTHK1-SUM          VRT 260
C
C      INITIIZE TI AND DDTEMP -          VRT 270
C      TI(II)=TT          VRT 280
C      DO 20 J=1,NNZ          VRT 290
C      20 DDTEMP(II,J)=0.00          VRT 300
C      30 CONTINUE          VRT 310
C
C      INITIIZE DTEMP -          VRT 320
C      DO 40 J=1,NNZ          VRT 330
C      40 DTEMP(II,J)=DDTEMP(II,J)          VRT 340
C
C      FACTOR FOR TIME-DERIVATIVE -          VRT 350
C      THETA=1.00          VRT 360
C
C      SEE COATS,ET.AL., SPE J (DEC 1974) 590 FOR 1-DIM EQUATION.          VRT 370
C      BOUNDARY CONDITIONS -          VRT 380
C      UPPER -          VRT 390
C      DTEMP(II,1)=0.00          VRT 400
C      LOWER -          VRT 410
C      DTEMP(II,NNZ)=TT-TI(II)          VRT 420
C
C      CALCULATE FINITE ELEMENT COEFFICIENTS FOR MATRIX EQUATION -          VRT 430
C      J=0          VRT 440
C
C      -----

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DO 50 I=2,NNL          VRT 530
J=J+1                  VRT 540
A1=DELZ(II,I-1)*COEF/(6.00*DELT)  VRT 550
C1=DELZ(II,I)*CUEF/(6.00*DELT)  VRT 560
C
      A(J)=A1-THETA/DELZ(II,I-1)  VRT 570
      B(J)=2.00*A1+THETA/DELZ(II,I-1)+2.00*C1+THETA/DELZ(II,I)  VRT 580
      C(J)=C1-THETA/DELZ(II,I)  VRT 590
C      RIGHT-HAND SIDE -  VRT 600
C      USE DTEMP FROM LAST SOLUTION AS INITIAL CONDITIONS  VRT 610
      D(J)=(A1+(1.00-THETA)/DELZ(II,I-1))*DTEMP(II,I-1)+(2.00*A1-(1.00-  VRT 620
      1.00*THETA)/DELZ(II,I-1)+2.00*C1-(1.00-THETA)/DELZ(II,I))*DTEMP(II,I)+VRT 630
      2*(C1+(1.00-THETA)/DELZ(II,I))*DTEMP(II,I+1)  VRT 640
      VRT 650
50 CONTINUE  VRT 660
C
C      SOLVE BY THOMAS METHOD (SEE CALIF BULL NO 63-4 SEPT, 1971 P 521)  VRT 670
C      NOTE THAT THERE ARE NNZ-2=J EQUATIONS AND J UNKNOWNs  VRT 680
C
C      UPPER TRIANGULARIZE -  VRT 690
      F(1)=B(1)  VRT 700
      D(1)=D(1)-A(1)*DTEMP(II,1)  VRT 710
      D(J)=D(J)-C(J)*DTEMP(II,NNZ)  VRT 720
      DO 60 I=2,J  VRT 730
      F(I)=B(I)-C(I-1)*A(I)/F(I-1)  VRT 740
60 D(I)=D(I)-A(I)*D(I-1)/F(I-1)  VRT 750
C
C      BACK SUBSTITUTE -  VRT 760
      DTEMP(II,J+1)=D(J)/F(J)  VRT 770
      J1=J-1  VRT 780
      DO 70 I=1,J1  VRT 790
70 DTEMP(II,J+1-I)=(D(J-I)-C(J-I)*DTEMP(II,J+2-I))/F(J-I)  VRT 800
C
C      DTEMP NOW CONTAINS NEW VALUES FOR TEMPERATURE DIFFERENCE  VRT 810
C
C      COMPUTE TRANSIENT CONDUCTIVE LEAKAGE  VRT 820
      CQ=-2.00*COND*(DTEMP(II,NNZ)-DTEMP(II,NNZ-1))/DELZ(II,NNL)  VRT 830
C
C      SAVE DTEMP  VRT 840
      DO 80 J=1,NNZ  VRT 850
80 DTEMP(II,J)=DTEMP(II,J)  VRT 860
      RETURN  VRT 870
      END  VRT 880

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C SUBROUTINE FORMEQ(A,R,NBBU,MBED) FEG 10
C ****=  
C IMPLICIT REAL*8(A-H,O-Z) FEG 20
C CALLED FROM MAIN FEG 30
C PURPOSE: TO FORM THE FINAL MATRIX EQUATION FEG 40
C ----- FEG 50
C DIMENSION A(NBBU,MBED), R(NBBU) FEG 60
C FEG 70
C FEG 80
C COMMON /INPUT/ PHI(20,10),XK(20,10),YR(20,10),P(20,10),H(20,10),X(FEG 90
1400),X1(400),NP(20,10),NPP(200,2),DX(20),DY(10),DZ(20,10),G(20,10)FEG 100
COMMON /CONTROL/ NK,NX,NXX,NY,NYY,NB,NBB,NT,DELT,TIME,PHFWT,DF,XKC,FEG 110
1COND,CUEF,BETA,IPRT FEG 120
COMMON /WORK/ AX(21,10),BX(21,10),EX(21,10),AY(20,11),BY(20,11),EY(FEG 130
1(20,11),TX(21,10),TXK(21,10),TY(20,11),TYK(20,11),XM(20,10),XMASS(FEG 140
220,10),EN(20,10),ENERGY(20,10),C(20,10),D(20,10),F(20,10),G(20,10)FEG 150
3,QH(20,10),XN(20,10),IM(20,10),DTP(20,10),DTH(20,10),PHIC(20,10),PFEG 160
4OLD(20,10) FEG 170
COMMON /EXTRA/ MBW,MBE FEG 180
C ----- FEG 190
C MP=(MBW+1)/2 FEG 200
C MP - MAIN DIAGONAL STORED VERTICALLY) FEG 210
C FEG 220
C INITIALIZE ARRAYS - FEG 230
DO 10 K=1,NBB FEG 240
DO 10 KK=1,MBW FEG 250
10 A(K,KK)=0.00 FEG 260
C FEG 270
C COMPUTE MATRIX - FEG 280
DO 50 K=1,NB FEG 290
I=NPP(K,1) FEG 300
J=NPP(K,2) FEG 310
C FEG 320
C MAIN DIAGONAL TERMS - FEG 330
C PRESSURE - FEG 340
A(2*K-1,MP)=-EX(I,J)-EY(I,J)-EX(I+1,J)-EY(I,J+1) FEG 350
C ENTHALPY - FEG 360
A(2*K,MP-1)=-AX(I,J)-AY(I,J)-AX(I+1,J)-AY(I,J+1) FEG 370
A(2*K,MP)=BX(I,J)-BY(I,J)-BX(I+1,J)-BY(I,J+1) FEG 380
C FEG 390
C OFF DIAGONAL TERMS - FEG 400
II=I FEG 410
IF (J.EQ.1) GO TO 20 FEG 420
JJ=J-1 FEG 430
NN=NP(II,JJ) FEG 440
IF (NN.EQ.0) GO TO 20 FEG 450
NC=MP+(NN-K)*2 FEG 460
A(2*K-1,NC)=EY(I,J) FEG 470
A(2*K,NC-1)=AY(I,J) FEG 480
A(2*K,NC)=BY(I,J) FEG 490
20 CONTINUE FEG 500
IF (J.EQ.NY) GO TO 30 FEG 510
JJ=J+1 FEG 520

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NN=NP(II,JJ)
IF (NN.EQ.0) GO TO 30
NC=MP+(NN-K)*2
A(2*K-1,NC)=EY(I,J+1)
A(2*K,NC-1)=AY(I,J+1)
A(2*K,NC)=BY(I,J+1)
30 JJ=J
IF (I.EQ.1) GO TO 40
II=I-1
NN=NP(II,JJ)
IF (NN.EQ.0) GO TO 40
NC=MP+(NN-K)*2
A(2*K-1,NC)=EX(I,J)
A(2*K,NC-1)=AX(I,J)
A(2*K,NC)=BX(I,J)
40 IF (I.EQ.NX) GO TO 50
II=I+1
NN=NP(II,JJ)
IF (NN.EQ.0) GO TO 50
NC=MP+(NN-K)*2
A(2*K-1,NC)=EX(I+1,J)
A(2*K,NC-1)=AX(I+1,J)
A(2*K,NC)=BX(I+1,J)
50 CONTINUE
C
C COMPUTE KNOWN VECTOR -
DO 70 K=1,NBB
II=K-MP
R(K)=0.00
DO 60 JJ=1,MBW
II=II+1
IF (II.LT.1) GO TO 60
IF (II.GT.NBB) GO TO 70
R(K)=R(K)+A(K,JJ)*X1(II)
60 CONTINUE
70 CONTINUE
C
DO 80 K=1,NB
I=NPP(K,1)
J=NPP(K,2)
C
C NEWTON-RAPHSON RESIDUAL VECTOR -
R(2*K-1)=-R(2*K-1)+XM(1,J)/DELT-XMASS(1,J)/DELT-G(I,J)
R(2*K)=-R(2*K)+EN(1,J)/DELT-ENERGY(I,J)/DELT-QH(I,J)
C
C LINEARIZED NEWTON-RAPHSON MATRIX -
A(2*K-1,MP)=A(2*K-1,MP)-F(I,J)*UX(I)*UZ(I,J)*DY(J)/DELT
A(2*K-1,MP+1)=A(2*K-1,MP+1)-G(I,J)*UX(I)*DY(J)*DZ(I,J)/DELT
A(2*K,MP-1)=A(2*K,MP-1)-C(I,J)*DX(I)*UY(J)*DZ(I,J)/DELT
A(2*K,MP)=A(2*K,MP)-D(I,J)*DX(I)*DY(J)*DZ(I,J)/DELT
80 CONTINUE
RETURN
END

```

FEG 530  
FEG 540  
FEG 550  
FEG 560  
FEG 570  
FEG 580  
FEG 590  
FEG 600  
FEG 610  
FEG 620  
FEG 630  
FEG 640  
FEG 650  
FEG 660  
FEG 670  
FEG 680  
FEG 690  
FEG 700  
FEG 710  
FEG 720  
FEG 730  
FEG 740  
FEG 750  
FEG 760  
FEG 770  
FEG 780  
FEG 790  
FEG 800  
FEG 810  
FEG 820  
FEG 830  
FEG 840  
FEG 850  
FEG 860  
FEG 870  
FEG 880  
FEG 890  
FEG 900  
FEG 910  
FEG 920  
FEG 930  
FEG 940  
FEG 950  
FEG 960  
FEG 970  
FEG 980  
FEG 990  
FEG 1000  
FEG 1010  
FEG 1020  
FEG 1030  
FEG 1040  
FEG 1050-

```

C      SUBROUTINE SOLVE (KKK,D,R,NEQ,IHALFB,NUDIM,MDIM)
C      *****
C      IMPLICIT REAL*8(A-H,O-Z)
C
C      ASYMMETRIC BAND MATRIX EQUATION SOLVER
C          ORIGINALLY PROGRAMED BY JAMES C. DUGUIC
C
C      KKK=1 TRIANGULARIZES THE BAND MATRIX B
C      KKK=2 SOLVES FOR RIGHT SIDE R, SOLUTION RETURNS IN R
C
C      DIMENSION B(NUDIM,NUDIM), R(NUDIM)
C      NRS=NEQ-1
C      IHBP=IHALFB+1
C      IF (KKK.EQ.2) GO TO 30
C
C      TRIANGULARIZE MATRIX A USING DOOLITTLE METHOD
C
C      DO 20 K=1,NRS
C          PIVOT=B(K,IHBP)
C          KK=K+1
C          KC=IHBP
C          DO 10 I=KK,NEQ
C              KC=KC-1
C              IF (KC.LE.0) GO TO 20
C              C=-B(I,KC)/PIVOT
C              B(I,KC)=C
C              KI=KC+1
C              LIM=KC+IHALFB
C              DO 10 J=KI,LIM
C                  JC=IHBP+J-KC
C                  10 B(I,J)=B(I,J)+C*B(K,JC)
C 20 CONTINUE
C          GO TO 100
C
C      MODIFY LOAD VECTOR R
C
C 30 NN=NEQ+1
C      IBAND=2*IHALFB+1
C      DO 70 I=2,NEQ
C          JC=IHBP-I+1
C          JI=1
C          IF (JC.LE.0) GO TO 40
C          GO TO 50
C 40 JC=1
C          JI=I-IHBP+1
C 50 SUM=0.0
C          DO 60 J=JC,IHALFB
C              SUM=SUM+B(I,J)*R(JI)
C 60 JI=JI+1
C 70 R(I)=R(I)+SUM
C
C      BACK SOLUTION

```

C

```
R(NEQ)=R(NEW)/B(NEW,IMBP)
DO 90 IBACK=2+NEQ
I=NN-IBACK
JP=I
KR=IHBP+1
MR=MINO(IBAND,IHALFB+1BACK)
SUM=0.0
DO 80 J=KR,MR
JP=JP+1
80 SUM=SUM+B(I,J)*R(JP)
90 R(I)=(R(I)-SUM)/B(I,IMBP)
100 RETURN
END
```

SOL 530
SOL 540
SOL 550
SOL 560
SOL 570
SOL 580
SOL 590
SOL 600
SOL 610
SOL 620
SOL 630
SOL 640
SOL 650
SOL 660-

```

C      SUBROUTINE PDATA(P,H,TIME,L,NY,NX)          PDT 10
C      #####*#
C      IMPLICIT REAL*8(A-H,O-Z)                  PDT 20
C
C      CALLED FROM MAIN                         PDT 30
C      PURPOSE: TO PRINT COMPUTED PRESSURE AND ENTHALPY PDT 40
C      -----
C      DIMENSION P(20,10), H(20,10)                PDT 50
C      -----
C      .
C      PRINT 30, L,TIME                          PDT 60
C      DO 10 J=1,NY                            PDT 70
C      10 PRINT 40, (P(I,J),I=1,NX)            PDT 80
C      PRINT 50                                PDT 90
C      DO 20 J=1,NY                            PDT 100
C      20 PRINT 40, (H(I,J),I=1,NX)            PDT 110
C      RETURN                                 PDT 120
C
C      30 FORMAT (///11X,'STEP NUMBER',I4,10X,'TIME',E10.3/11X,15(1H*)///11XPDT 130
C           1,'PRESSURE VALUES'/11X,15(1H-)//)          PDT 140
C      40 FORMAT (/(11X,8(G12.5,2X)))           PDT 150
C      50 FORMAT (///11X,'ENTHALPY VALUES'/11X,15(1H-)//) PDT 160
C      END                                     PDT 170
C                                              PDT 180
C                                              PDT 190
C                                              PDT 200
C                                              PDT 210
C                                              PDT 220
C                                              PDT 230-

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C      SUBROUTINE PHREG(INDEX)          PHR 05
C      *****
C      IMPLICIT REAL*8(A-H,O-Z)          PHR 10
C
C      CALLED FROM MAIN                 PHR 25
C      PURPOSE: TO DETERMINE WHAT THERMODYNAMIC REGION A
C              FINITE-DIFFERENCE BLOCK IS IN          PHR 40
C      -----
C      DIMENSION INDEX(200)             PHR 50
C                                         PHR 60
C                                         PHR 70
C                                         PHR 80
C
C      COMMON /INPUT/ PHI(20,10),XK(20,10),YK(20,10),P(20,10),H(20,10),X(PHR 90
C      1400),XI(400),NP(20,10),NPP(200,2),DX(20),DY(10),DZ(20,10),Q(20,10)PHR 100
C      COMMON /CONTROL/ NK,NX,NXX,NY,NYY,NB,NBB,NT,DELT,TIME,PHFWT,DF,XKC,PHR 110
C      ICOND,COEF,BETA,IPRT           PHR 120
C
C      -----
C      INDEX=-1 FOR COMPRESSED WATER      PHR 130
C      INDEX=0  FOR TWO-PHASE            PHR 140
C      INDEX=1  FOR SUPERHEATED STEAM     PHR 150
C      DO 10 K=1,NB                      PHR 160
C      PP=X1(2*K-1)*0.1**7               PHR 170
C      HH=X1(2*K)*0.1**7                PHR 180
C
C      COMPUTE SATURATED STEAM (HS) AND SATURATED WATER (HW) ENTHALPY - PHR 200
C      HS=2822.82-39.952/PP+2.54342/PP/PP-0.938879*PP*PP
C      HW=730.984+129.239*PP-10.0333*PP*PP+0.39881*PP*PP*PP
C      1  -99.0697/PP+12.9267/PP/PP-0.628359/PP/PP/PP
C      INDEX(K)=0                         PHR 240
C      IF (HH.LT.HW) INDEX(K)=-1          PHR 250
C      IF (HH.GT.HS) INDEX(K)=1          PHR 260
C
C      IF TEMPERATURE IS BELOW 50 DEG C (H=209.33) ASSUME IN COMP WATER PHR 270
C      IF (HH.LT.209.33D0) INDEX(K)=-1
C
C      10 CONTINUE
C      RETURN
C      END

```

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SUBROUTINE BALNCE(ICX,KKK,PCEE)                                BAL  05
C ****
C IMPLICIT REAL*8(A-H,O-Z)                                     BAL 10
C
C CALLED FROM MAIN                                             BAL 20
C PURPOSE: TO COMPUTE MASS AND ENERGY BALANCE                  BAL 40
C -----
C COMMON /INPUT/ PHI(20,10),XK(20,10),YK(20,10),P(20,10),H(20,10),X(BAL 70
1400),X1(400),NP(20,10),NPP(200,2),DX(20),DY(10),DZ(20,10),Q(20,10)BAL 80
COMMON /CONTROL/ NK,NX,NXX,NY,NYY,NB,NBB,NT,DELT,TIME,PHFWT,DF,XKC,BAL 90
ICOND,COEF,BETA,IPRT                                         BAL 100
COMMON /WORK/ AX(21,10),BX(21,10),EX(21,10),AY(20,11),BY(20,11),EYBAL 110
1(20,11),TX(21,10),TXK(21,10),TY(20,11),TYK(20,11),XM(20,10),XMASS(BAL 120
220,10),EN(20,10),ENERGY(20,10),C(20,10),D(20,10),F(20,10),G(20,10)BAL 130
3,QH(20,10),XN(20,10),TM(20,10),DTP(20,10),DTH(20,10),PHIO(20,10),PBAL 140
4OLD(20,10)                                                 BAL 150
COMMON /CHECK/ IND(200),INDOLD(200)                           BAL 160
C -----
C STATEMENT FUNCTIONS FOR THERMODYNAMIC PROPERTIES -          BAL 170
C
C F1(PX,HX)=1.00207+4.42607D-4*PX-5.47456D-5*HX+5.02875D-7*HX*PX-1.2BAL 210
14791D-7*HX*HX                                         BAL 220
F2(PX,HX)=-2.41231+2.56222D-1*HX-9.31415D-3*PX*PX-2.2568D-5*HX*HX BAL 230
F3(PX,HX)=-2.26162D-5+0.0438441D0*PX-1.79088D-5*PX*HX+3.69276D-8*PBAL 240
1X*PX*PX*PX+5.17644D-13*HX*HX*HX*PX                   BAL 250
F4(PX,HX)=-374.669D0+47.9921D0*PX-0.633606D0*PX*PX+7.39386D-5*HX*HXBAL 260
1X-3.3372D6/HX/HX/PX/PX+0.0357154D0/PX/PX/PX-1.1725D-9*HX*HX*HX*PX-BAL 270
22.26861D15/HX/HX/HX/HX                               BAL 280
C
C DELM=0.D0                                              BAL 290
C DELE=0.D0                                              BAL 300
DO 70 I=1,NX                                            BAL 310
DO 70 J=1,NY                                            BAL 320
IF (DZ(I,J).GT.0.D0) GO TO 10                           BAL 330
XM(I,J)=0.D0                                           BAL 340
EN(I,J)=0.D0                                           BAL 350
GO TO 70                                              BAL 360
10 CONTINUE                                              BAL 370
PP=P(I,J)                                              BAL 380
HH=H(I,J)                                              BAL 390
PP=PP*0.1**7                                           BAL 400
HH=HH*0.1**7                                           BAL 410
K=NP(I,J)                                              BAL 420
PHI(I,J)=PHIO(I,J)*(1.0D0+(P(I,J)-POLD(I,J))*BETA)  BAL 430
HW=730.984+129.239*PP-10.0333*PP*PP+0.39881*PP*PP*PP
1 -99.0697/PP+12.9267/PP/PP-0.628359/PP/PP/PP
IF (IND(K)) 20,30,40                                  BAL 470
20 CONTINUE                                              BAL 480
DEN=F1(PP,HH)                                           BAL 490
TEMP=F2(PP,HH)                                           BAL 500
GO TO 50                                              BAL 510

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30 CONTINUE                                BAL 520
    HS=2822.82-39.952/PP+2.54342/PP/PP-0.938879*PP*PP
    TEMP=F2(PP,HW)                            BAL 530
    DW=F1(PP,HW)                            BAL 540
    DS=F3(PP,HS)                            BAL 550
    SW=DS*(HS-HH)/(HH*(DW-US)-(HW*DW-HS*DS))  BAL 560
    SST=1.0-SW                             BAL 570
    DEN=DW*SW+SST*DS                        BAL 580
    GO TO 50                                BAL 590
40 CONTINUE                                BAL 600
    DEN=F3(PP,HH)                            BAL 610
    HW=730.984+129.239*PP-10.0333*PP*PP+0.39881*PP*PP*PP
    1 -99.0697/PP+12.9267/PP/PP-0.628359/PP/PP/PP
    HS=2822.82-39.952/PP+2.54342/PP/PP-0.938879*PP*PP
    TT1=F4(PP,HH)                            BAL 620
    TT2=F4(PP,HS)
    TT3=F2(PP,HW)
    TEMP=TT1-TT2+TT3
50 HROCK=TEMP*PHFWT                         BAL 640
    VOL=DY(J)*DX(I)*DZ(I,J)
    XMT=VOL*DEN*PHI(I,J)
    ENT=VOL*(PHI(I,J)*HH*DEN*10.**7+(1.0-PMI(I,J))*HROCK*DF)
    IF (ICX.EQ.0) GO TO 60
    DELM=DELM-XMASS(I,J)+XMT
    DELE=DELE-ENERGY(I,J)+ENT
60 XM(I,J)=XMT                             BAL 650
    EN(I,J)=ENT                            BAL 660
70 CONTINUE                                BAL 670
    IF (KKK.NE.NK) GO TO 90
    DO 80 I=1,NX                            BAL 680
    DO 80 J=1,NY                            BAL 690
    XMASS(I,J)=XM(I,J)                      BAL 700
80 ENERGY(I,J)=EN(I,J)                      BAL 710
90 CONTINUE                                BAL 720
    IF (ICX.EQ.0) GO TO 110
    QQQ=0.D0                               BAL 730
    EEE=0.D0                               BAL 740
    DO 100 I=1,NX                           BAL 750
    DO 100 J=1,NY                           BAL 760
    QQQ=QQQ+Q(I,J)*DELT                  BAL 770
100 EEE=EEE+QH(I,J)*DELT                  BAL 780
    PCEM=(DELM-QQQ)*100.D0                BAL 790
    IF (QQQ.NE.0.D0) PCEM=PCEM/QQQ        BAL 800
    PCEE=(DELE-EEE)*100.D0                BAL 810
    IF (EEE.NE.0.D0) PCEE=PCEE/EEE        BAL 820
    WRITE (6,120) QQQ,EEE,DELM,DELE,PCEM,PCEE
110 CONTINUE                                BAL 830
    RETURN                                  BAL 840
C
120 FORMAT (//11X,23HMASS AND ENERGY BALANCE/11X,23(1H-)/32X,4HMASS,31BAL 950
1X,4HHEAT/15X,12HDISCHARGE = ,G15.8,20X,G15.8/15X,12HSTORAGE = ,GBAL 960
215.8,20X,G15.8/15X,12H* ERROR = ,G15.8,20X,G15.8)  BAL 970
    END                                     BAL 980-

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